

GEOMETRICAL OPTICS

(<http://www.a-levelphysicstutor.com/optics-convx-lnss.php>)

Convex lens

Concave lens

Plane mirror

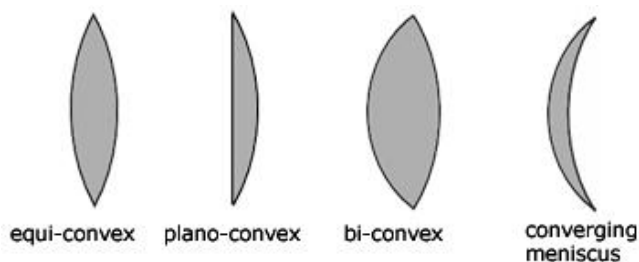
Convex mirror

Concave mirror

Convex Lenses

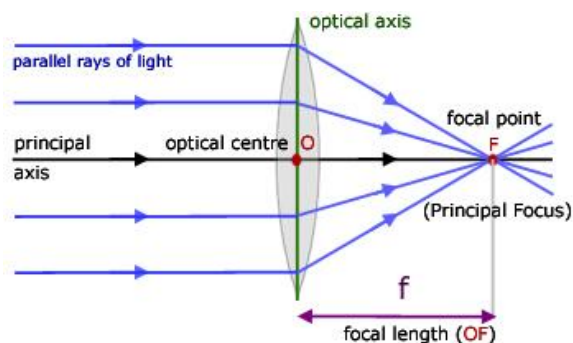
types	basic diagram	power	ray diagrams
lens formula	magnification	chromatic aberration	

Types of lenses



All four types of convex lens are converging lenses. That is, they bring parallel rays of light to a focus, producing a real image.

Basic Ray Diagram



The basic ray diagram for a convex lens introduces a number of important terms:

principal axis - the line passing through the centres of curvature of the lens

principal focus - a point on the principal axis where rays of light parallel to the principal axis converge

focal length - the horizontal distance between the principal focus and the optical centre of the lens

optical centre - an imaginary point inside a lens through which a light ray is able to travel without being deviated

centre of curvature - the centre of the sphere of which the lens surface is a part

Power of a Lens

$$P = \frac{1}{f}$$

The power **P** of a lens is the inverse of its focal length **f**. Since **f** is measured in metres '**m**' the units of lens power are **m⁻¹**.

The power also depends on the type of lens.

Convex lenses have **positive** powers.

Concave lenses all have **negative** powers.

For example, a 10 cm focal length convex lens has a power of +10 m⁻¹; while a 20 cm focal length concave lens has a power of -5 m⁻¹.

Ray diagrams

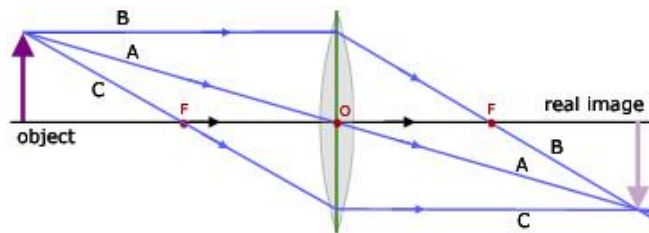
To understand ray diagrams it is important to know something about images. Images come in two categories :

real images - are produced from actual rays of light coming to a focus (eg a film projected onto a screen)

virtual images - are produced from where rays of light appear to be coming from (eg a magnifying glass image)

Ray diagrams are constructed by taking the path of **three** distinct rays **from a point on the object**:

note - the lens is considered to be so thin as to be represented by the axis of the lens(green)



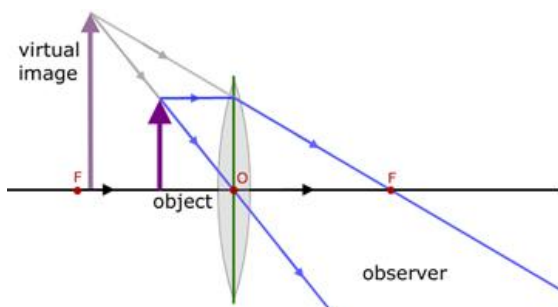
A) a ray passing through the optical centre of the lens

B) a ray parallel to the principal axis, which refracts through the lens, passing through the principal focus

C) a ray passing through the principal focus(on the same side as the object) and being refracted through the lens, emerging parallel to the principal axis.

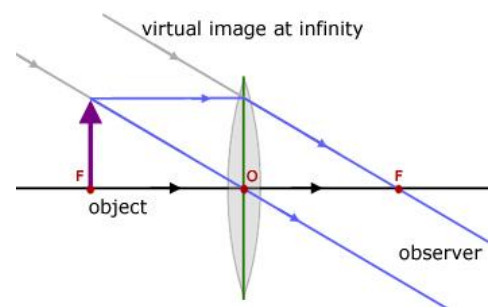
The diagrams(below) represent the formation of an image from :

object between f and the lens (magnifying glass)



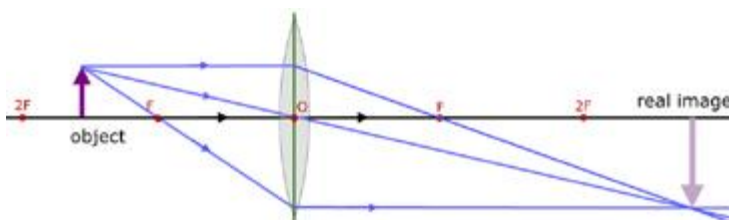
The image is the same side of the lens as the object and is upright, virtual and magnified.

object at f



The image is formed at infinity from parallel rays that do not converge. Therefore no image is formed.

object between f and $2f$ (projector)



The image is on the opposite side of the lens than the object. It is real, inverted and magnified.

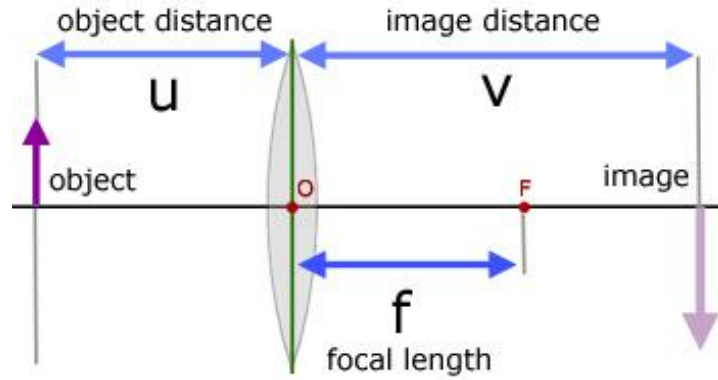
object at $2f$

The diagram is very similar to the left one, showing the three construction rays. This time the image is the same distance **behind** the lens as the object is **in front**. As before, the image is inverted and real, but is the same size as the object.

object at infinity

The image is formed at the focal point of the lens. It is real, inverted and diminished in size.

The Lens Formula



$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

When using this equation a sign convention must be obeyed:

Distances from lenses to real objects & real images are positive.

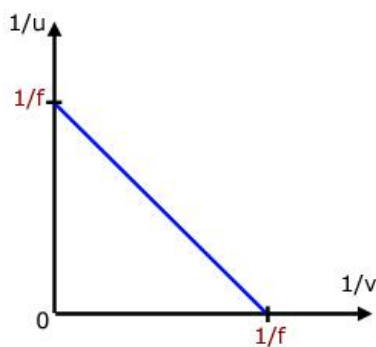
Distances from lenses to imaginary objects and imaginary images are negative.

Focal lengths of convex lenses are positive.

Focal lengths of concave lenses are negative.

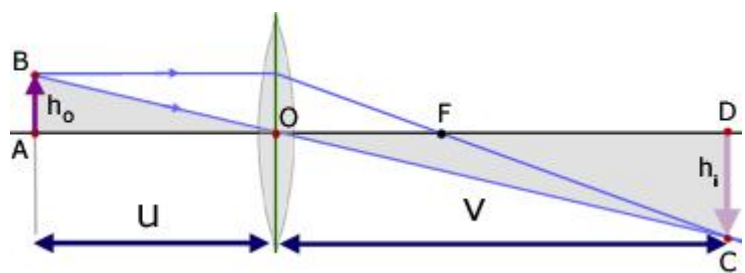
The focal length f of a lens can be found quite accurately by moving an illuminated object in front of a lens so that an image is cast on a screen.

By taking readings of image and object distances for different positions, a graph can be drawn.



Plotting $1/u$ against $1/v$ gives a straight line with a negative gradient. The focal length can be found from the intercept, which is $1/f$ on both the x and y axes.

Magnification



Magnification (m) is simply the image height divided by the object height.

$$m = \frac{h_i}{h_o}$$

To obtain a relation involving object distance (u) and image distance (v), consider the image formed by two rays from a point on the object.

Triangles **AOB** and **COD** are similar.

Therefore,

$$\frac{CD}{AB} = \frac{DO}{AO} \quad \frac{h_i}{h_o} = \frac{v}{u} \quad \underline{m = \frac{v}{u}}$$

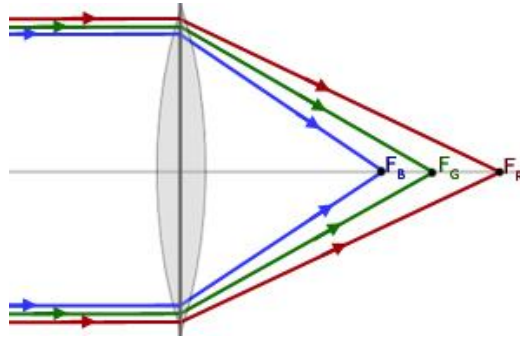
Chromatic Aberration

Chromatic aberration is the dispersion of white light by a convex lens.

The different coloured components of white light are brought to different foci according to their wavelength.

Since 'red rays refract least', red light produces the longest wavelength and violet the shortest.

In the diagram, F_B , F_G , F_R are the principal foci for blue, green and red light.



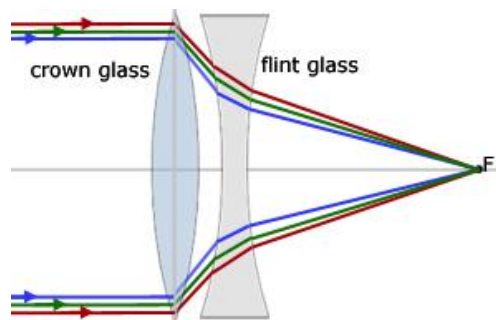
Chromatic aberration affects the image by making it appear blurred with fringes of colour around it. This is a result of only one colour being in focus at a time.

Chromatic aberration can be corrected using a **chromatic doublet**.

This is a combination of two lenses, one convex and the other concave.

The lenses are of different types of glass (crown glass & flint glass) .

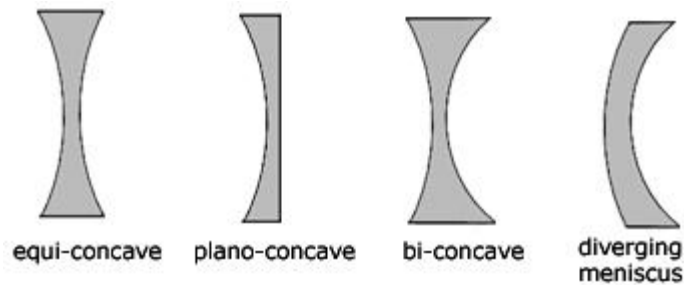
The pair are cemented together (not shown) using Canada Balsam glue. This has a refractive index mid-way between the two glass types. In any event, the glue layer is extremely thin and has little effect.



Concave Lenses

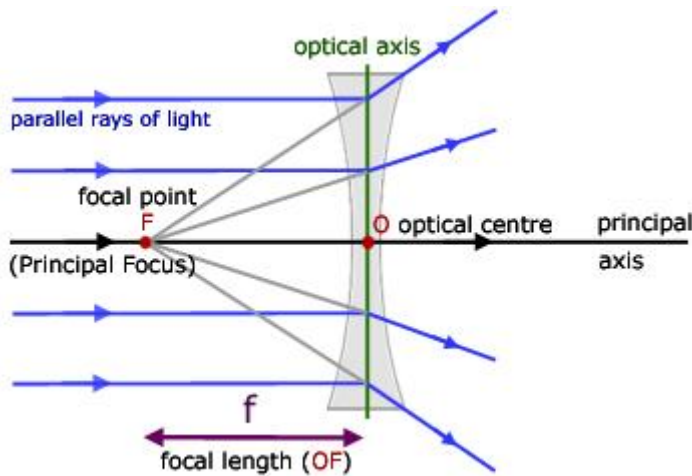
types	basic diagram	ray diagrams
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Types of Lens



All four types of concave lens are diverging lenses. That is, they diverge parallel rays of light from a focus, producing a virtual image.

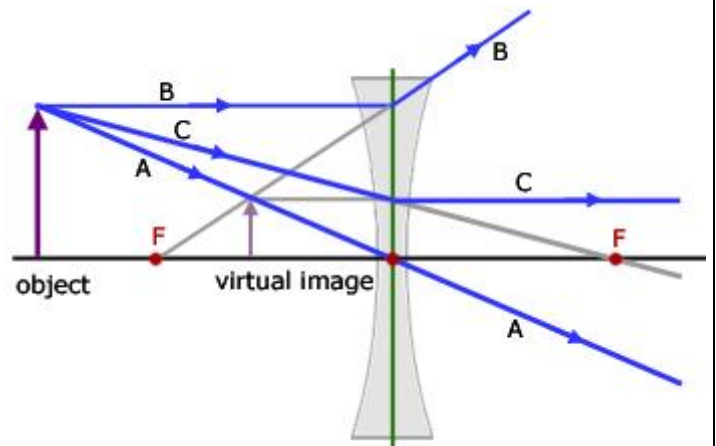
Basic Ray Diagram



Ray Diagrams

Ray diagrams are constructed by taking the path of **three** distinct rays **from a point on the object** :

note - the lens is considered to be so thin as to be represented by the axis of the lens (green)



A) a ray passing through the optical centre of the lens

B) a ray parallel to the principal axis, which refracts through the lens and appears to have come from the principal focus

C) a ray heading towards the principal focus (on the opposite side of the lens) and being refracted through the lens, emerging parallel to the principal axis

For all the object positions listed below:

- object between f and the lens**
- object at f**
- object between f and $2f$**
- object at $2f$**
- object at infinity**

the ray diagrams are virtually the same as in the diagram above. Hence the result is the same.

The image produced is **virtual, upright, diminished and on the same side of the lens as the object.**

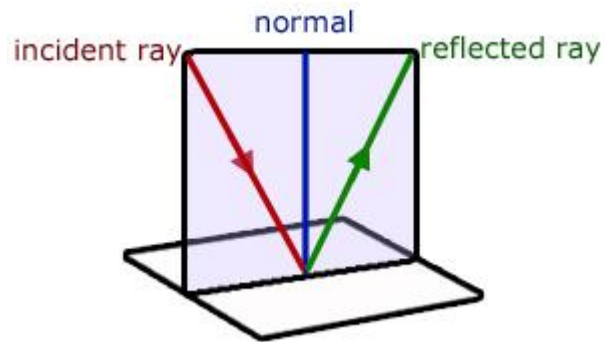
Plane Mirrors

laws of reflection

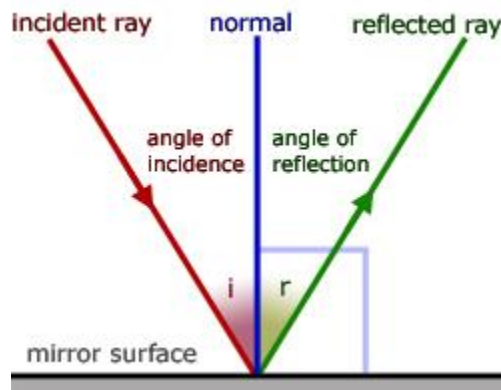
images

mirror rotation

Laws of Reflection

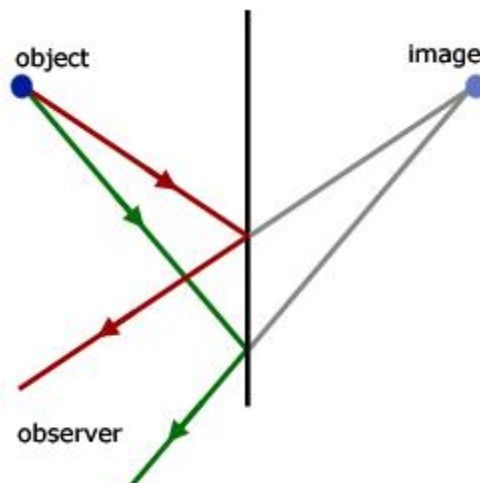


1. **The incident ray, the reflected ray and the normal, at the point of incidence, all lie in the same plane.**

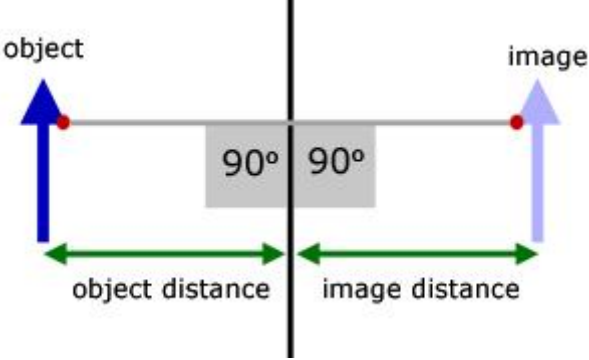



2. **The angle of reflection equals the angle of incidence.**

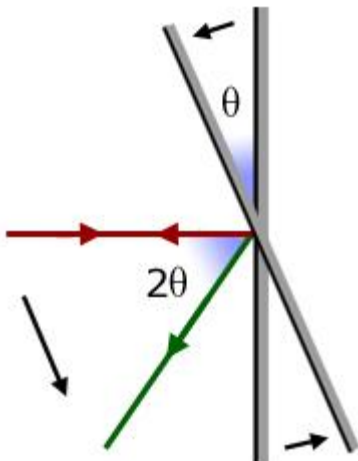
Hence an image can be located by taking two light rays from a point object and retracing them after reflection.



Plane Mirror Images

	<ol style="list-style-type: none">1. All images are virtual. That is, they cannot be projected on to a screen.2. The image produced in a mirror is as far behind the mirror as the object is in front. object distance = image distance3. The image is the same size as the object.4. A line joining a point on the image to a corresponding point on the object is perpendicular to the mirror.
	<ol style="list-style-type: none">5. The image is laterally inverted (sideways upside down).

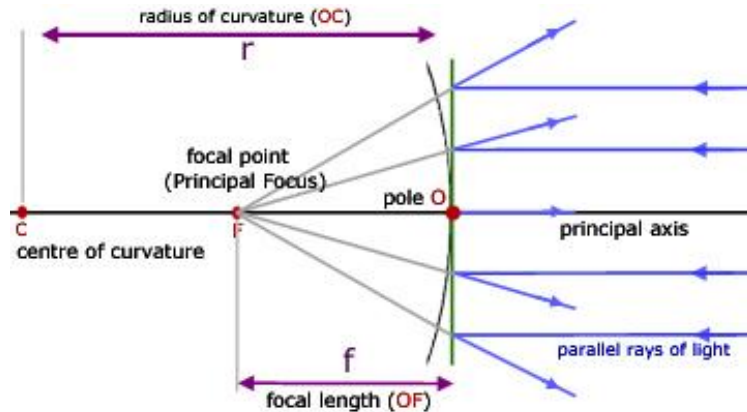
Mirror Rotation

	<p>When a light ray is normal to a mirror it reflects back the way it came.</p> <p>However, if the mirror is tilted say, through θ° (theta) then the reflected light ray rotates through $2\theta^\circ$.</p>
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Convex Mirrors

basic diagram	ray diagrams	proof $r = 2f$
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Basic Ray Diagram



The basic ray diagram for a convex mirror introduces a number of important terms:

aperture - the diameter of the circular mirror

pole - where the principal axis meets the mirror surface

centre of curvature - the centre of the sphere that the mirror forms part

radius of curvature (r) - radius of the sphere

principal axis - the line through the centre of curvature and the pole of the mirror

focal length (f) - equal to half the radius of curvature $f = r/2$

Ray Diagrams

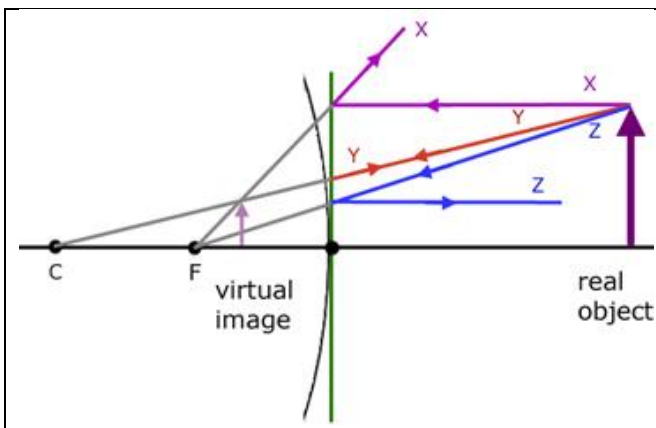
Ray diagrams are constructed by taking the path of three distinct rays from a point on the object:

X) a ray parallel to the principal axis reflected through **F** (the principal focus)

Y) a ray passing through **C** which is then reflected back along its original path

Z) a ray passing through F, which is then reflected parallel to the principal axis

note - the convex mirror is considered to be so thin as to be represented by a vertical line



For all the object positions listed below,

object between f and the mirror

object at f

object between f and $2f$

object at $2f$

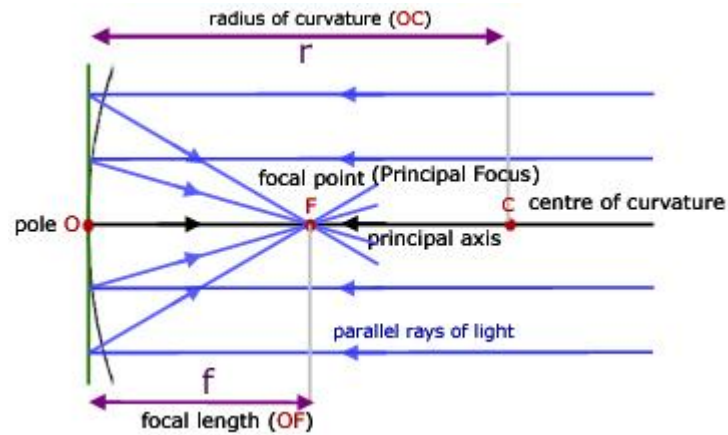
object at infinity

the ray diagrams are virtually the same as in the diagram on the left. Hence the result is the same. The image produced is virtual, upright and diminished.

Concave Mirrors

basic diagram	ray diagrams	proof of $r = 2f$
mirror formula	caustic curves	parabolic mirrors

Basic Ray Diagram



The basic ray diagram for a concave mirror introduces a number of important terms:

aperture - the diameter of the circular mirror

pole - where the principal axis meets the mirror surface

centre of curvature - the centre of the sphere that the mirror forms part

radius of curvature (r) - radius of the sphere

principal axis - the line through the centre of curvature and the pole of the mirror

focal length (f) - equal to half the radius of curvature $f = r/2$

Ray Diagrams

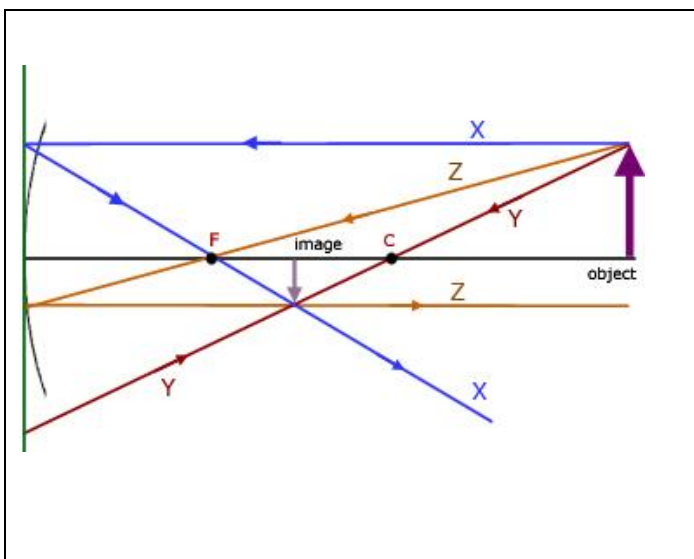
Ray diagrams are constructed by taking the path of three distinct rays from a point on the object:

X) a ray parallel to the principal axis reflected through **F** (the principal focus)

Y) a ray passing through **C** which is then reflected back along its original path

Z) a ray passing through **F**, which is then reflected parallel to the principal axis

note - the concave mirror is considered to be so thin as to be represented by a vertical line



The image types and positions for a concave mirror are very similar to those of a convex lens.

object between F and the mirror

This is similar to a magnifying glass, with a practical use as a make-up mirror.

object at F

The image is formed at infinity from parallel rays that do not converge. Therefore no image is formed.

object between F and C

The image is real, inverted and magnified.

object at C

The image is formed at C.

The image is inverted, real and the same size as the object.

object at infinity

The image is formed at the focal point of the lens. It is real, inverted and diminished in size.

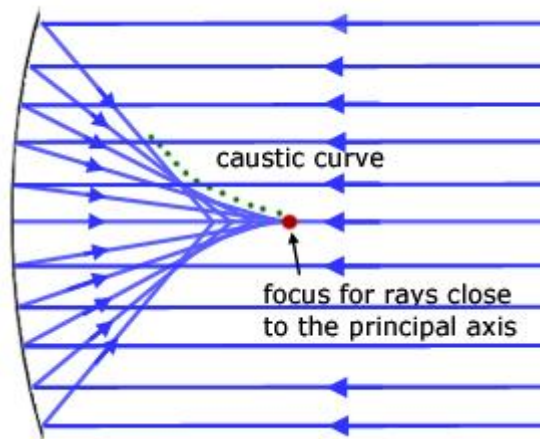
Proof of $r = 2f$

Convex mirror	Concave mirror
<p>The angles in the triangle(CFX) are vertically opposite and corresponding angles from the angles of reflection and incidence.</p> <p>From the diagram it is apparent that XF does not equal PF.</p> <p>However, when point X is closer to the principal axis, XF approaches the size of PF.</p> <p>So for rays close to the principal axis, we can say that XF = PF.</p> <p>In other words,</p> $\mathbf{XC = PF + FC}$ $\mathbf{r = f + f}$ $\mathbf{r = 2f}$	<p>The angle of incidence equals the angle at C (corresponding angles).</p> <p>From the diagram it is apparent that XF does not equal PF.</p> <p>However, when point X is closer to the principal axis, XF approaches the size of PF.</p> <p>So for rays close to the principal axis we can say that XF = PF.</p> <p>In other words,</p> $\mathbf{XC = PF + FC}$ $\mathbf{r = f + f}$ $\mathbf{r = 2f}$

The Mirror Formula

$f = \frac{r}{2} \quad \frac{1}{f} = \frac{2}{r}$ $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ $\therefore \frac{1}{u} + \frac{1}{v} = \frac{2}{r}$	<p>The <i>Gaussian</i> sign convention, 'real is positive' is used:</p> <ol style="list-style-type: none"> 1) focal length (f) and radius of curvature (r) are both positive for concave mirrors 2) distances to real images and real objects are positive 3) distances to virtual images and virtual objects are negative
	<p>The <i>Cartesian</i> sign convention:</p> <ol style="list-style-type: none"> 1) The mirror is placed at the origin of the XY-coordinate plane, 2) The light direction is from left to right, 3) Any ray starts at origin, it is positive to the right, negative to the left. Hence, concave mirror has <i>negative focal length</i> and <i>negative radius of curvature</i> .

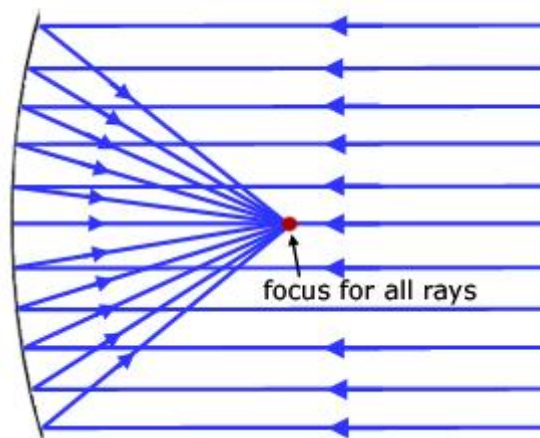
Caustic Curves



The **caustic** is the name given to the region of a concave mirror where parallel rays of light come to different foci.

This happens for **rays away** from the principal axis. The further away they are, the closer is the focus to the mirror.

Parabolic Mirrors



A parabolic mirror produces one focus for all rays parallel to the principal axis, irrespective of their distance from it.

Parabolic mirrors have uses in telescopes, solar furnaces, and car headlights/torches/floodlights etc.