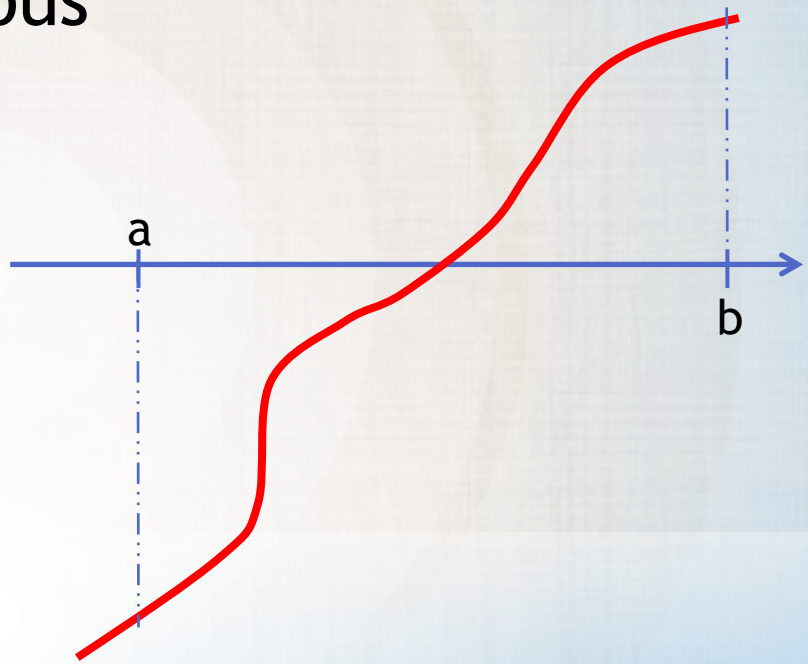
# Root-finding methods

$f(x) = 0$

$f(x)$ is defined and continuous




$f(a)*f(b)<0$

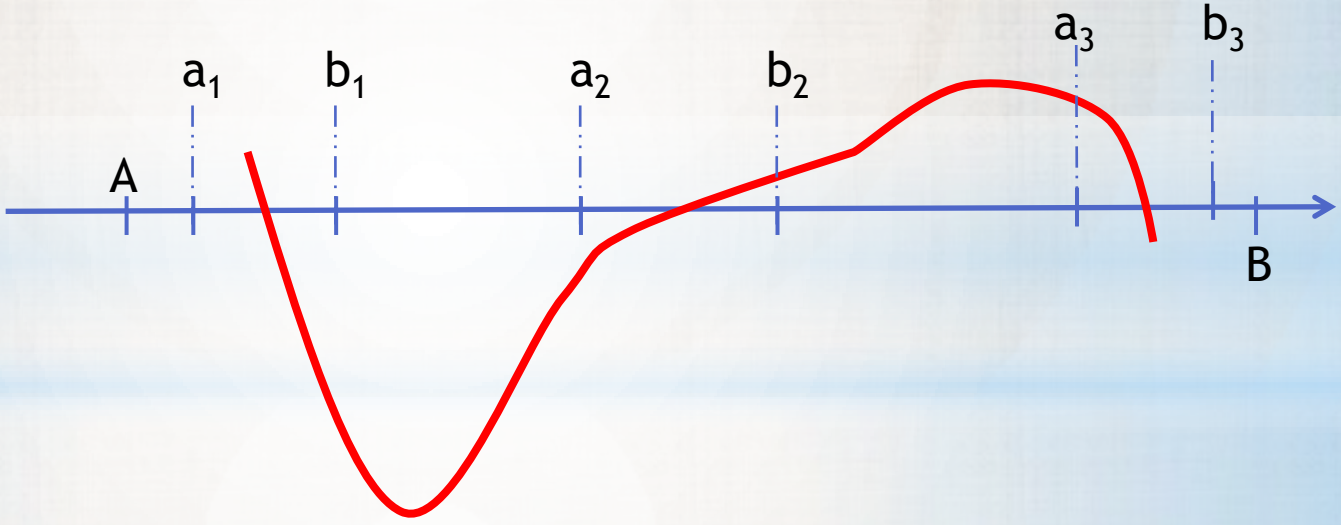
$f'(x)$ does not change sign

1. Separation (isolation) of the roots

$f(x) = 0$

$f(x)$ is defined and continuous

$f(a_i)*f(b_i)<0$



## 2. Determination (refinement) of the roots – root-finding

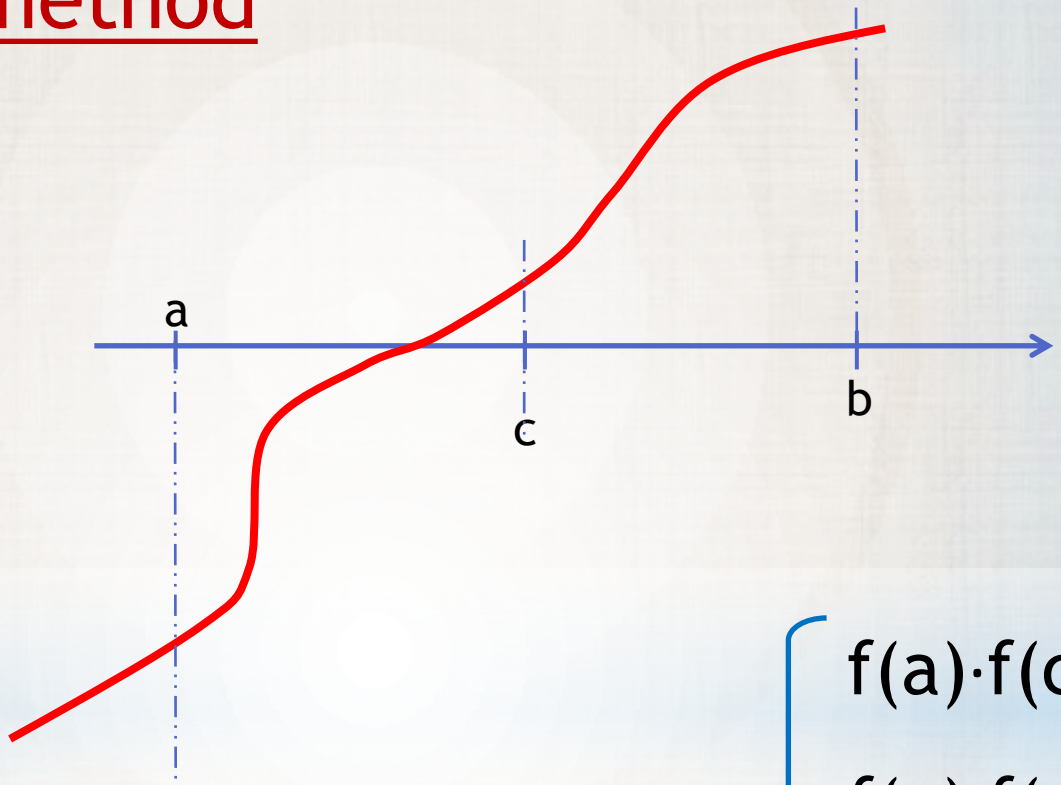
- bisectional method
- tangent line (Newton-Raphson) method
- secant line method
- combined tangent-and-secant line method
- combined bisectional-and secant line method

  (*aka* false position method)
- fixed point method (*aka* simple iteration method)
- ……

# Bisectional method

$f(x) = 0$

$f(a) \cdot f(b) < 0$




$f(a) \cdot f(c) = 0$ ?

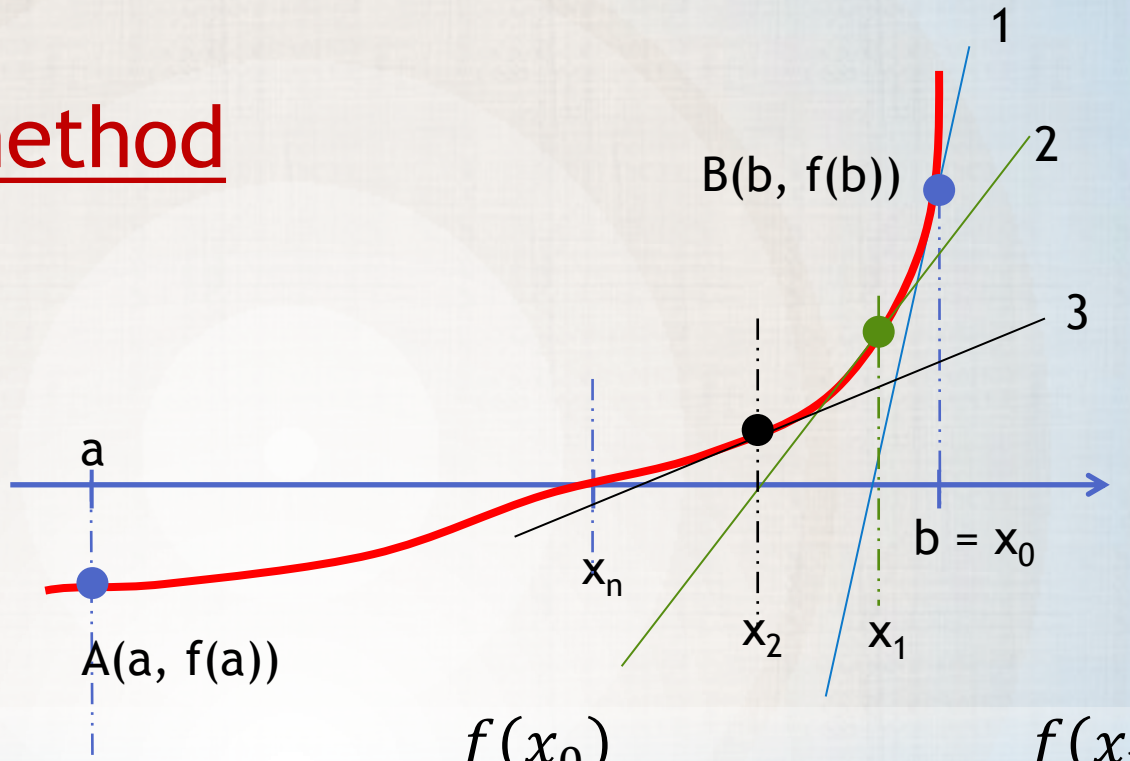
$f(a) \cdot f(c) < 0$ ?

$f(c) \cdot f(b) < 0$ ?

# Tangent line method

$f(x) = 0$

Tangent line eq.:
$y(x) = k \cdot x + m$
*At B:*
$k = f'(b)$
$m = f(b) - k \cdot b$




$$y(x) = f'(b) \cdot x + f(b) - f'(b) \cdot b$$

$$y(x_1) = f'(b) \cdot x_1 + f(b) - f'(b) \cdot b = 0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

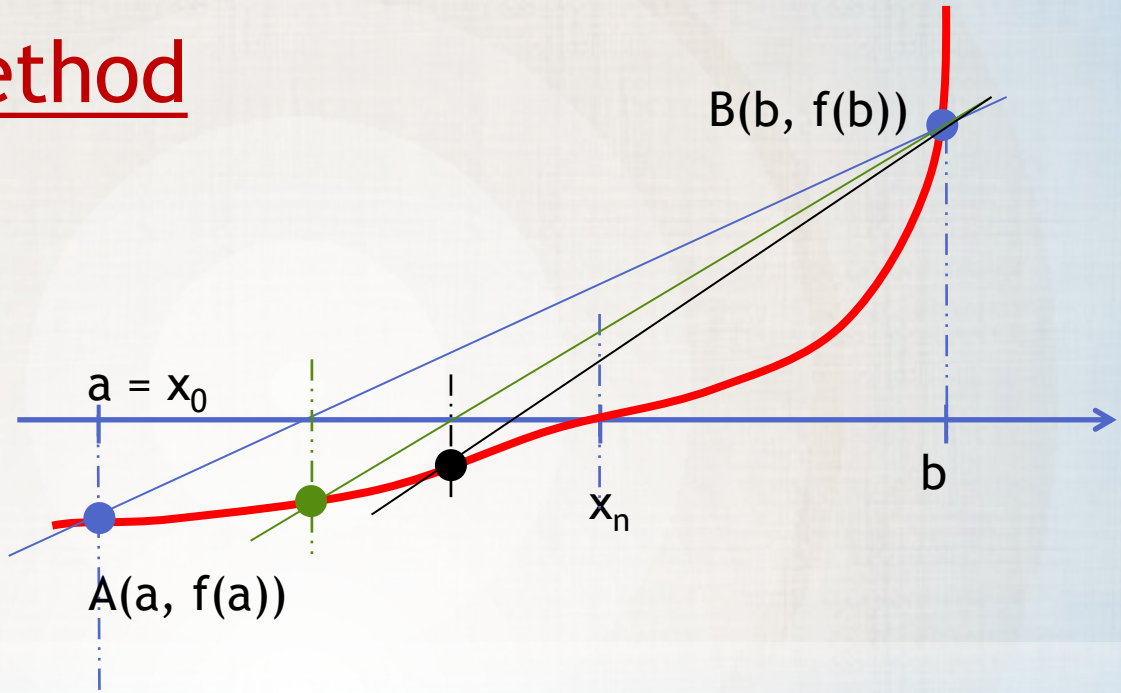
$f(a) \cdot f''(a) > 0 \Rightarrow x_0 := a$
$otherwise\ x_0 := b$

# Secant line method

$$f(x) = 0$$

*Line eq.:*

$$\frac{y - y_2}{y_2 - y_1} = \frac{x - x_2}{x_2 - x_1}$$



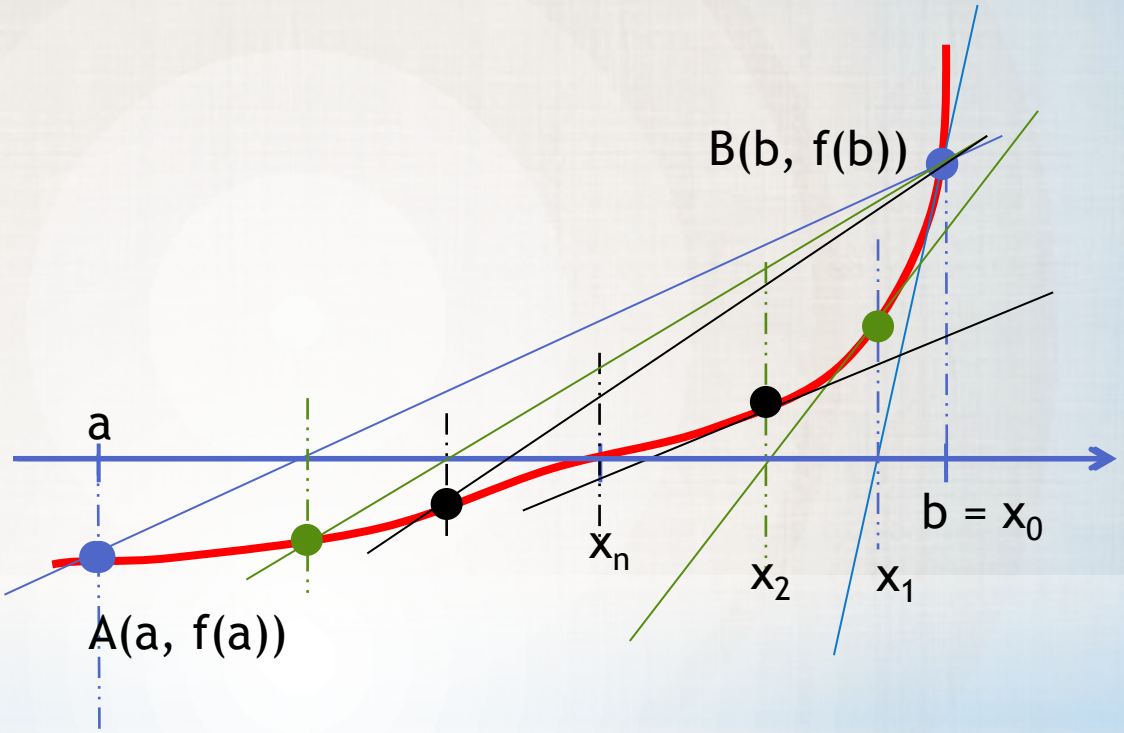

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f(b) - f(x_{n-1})} \cdot (b - x_{n-1})$$

$$f(a) \cdot f''(a) > 0 \;\Rightarrow\; x_0 := b;\; a \text{ is fixed}$$

$$f(b) \cdot f''(b) > 0 \;\Rightarrow\; x_0 := a;\; b \text{ is fixed}$$

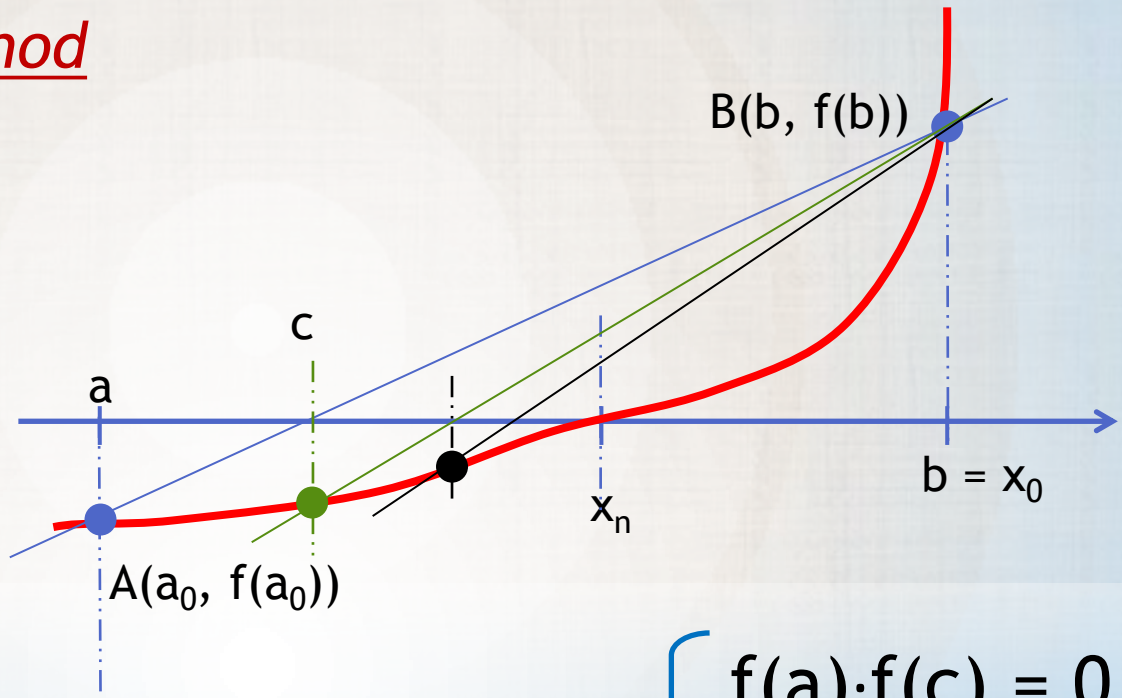
# Combined tangent-and-secant line method

$f(x) = 0$



# Combined secant line and bisectional method

*aka false position method*




$f(a)\cdot f(c) = 0$ ?

$f(a)\cdot f(c) < 0$ ?

$f(c)\cdot f(b) < 0$ ?

# Fixed point method

## aka simple iterations method

$f(x) = 0$

$\Downarrow$

$x = \phi(x)$

$\Downarrow$

$x_0 = guess\ value$

$\Downarrow$

$x_1 = \phi(x_0)$

$\Downarrow$

$x_2 = \phi(x_1)$

$\Downarrow$

$x_3 = \phi(x_2)$

$\Downarrow$

$\vdots$

$\Downarrow$

$x_n = \phi(x_{n-1})$

## Example

$f(x) := (x+1)^3 - x$

$g1(x) := (x+1)^3$ $\qquad$ $g2(x) := \sqrt[3]{x} - 1$

$x_1 := 0$ $\qquad$ $y_1 := 0$

$n := [2..10]$

$x_n := g1(x_{n-1})$ $\qquad$ $y_n := g2(y_{n-1})$

$x =$

$$y = \begin{bmatrix} 0 \\ -1 \\ -2 \\ -2.2599 \\ -2.3123 \\ -2.3224 \\ -2.3243 \\ -2.3246 \\ -2.3247 \\ -2.3247 \end{bmatrix}$$

$f(y_{10}) = 1.313 \cdot 10^{-5}$