PRACTICAL 7

DETERMINATION OF C_p/C_V OF GASES BY MEASURING THE SPEED OF SOUND

Objective: to determine the heat capacity ratio C_p/C_V for various gases by measuring the speed of sound by the method of superposition of mutually perpendicular oscillations.

Equipment & accessories: a sound generator, an electronic oscilloscope, an amplifier U2-6, a tube with a built-in microphone and a telephone, a pump, a pressure gauge, a camera with helium gas.

INTRODUCTION

There is a well-known method for the determination of C_p/C_V of gases is based on adiabatic expansion – the method of Clément-Desormes. However, only with perfect thermal isolation the gas expansion will be adiabatic. In practice, it is not so easy to eliminate the influx of the heat. It is more easy to achieve that with very fast flowing processes. Such processes occur in sound waves, the propagation velocity of which depends on the value of the adiabatic constant $\gamma = C_p/C_V$. This is the basis of the second, more accurate method of experimental determination the ratio of heat capacities for gases.

In gases and liquids that possess only bulk elasticity, but not elasticity of form, only longitudinal perturbations can propagate. The speed of propagation of longitudinal waves in liquids and gases is determined by the ratio:

$$\text{v} = \sqrt{\frac{dp}{d\rho}}, \qquad (1)$$

where p is a pressure, ρ is a mass density.

A sound wave in a gas is a process of sequential compression and rarefaction of the medium, and these processes occur very quickly, and the thermal conductivity of the gas is small. Therefore, the process of sound propagation in a gas should be considered as adiabatic. According to Poisson's equation, the adiabatic process for an ideal gas is described by the equation $pV^{\gamma} = const$. If, instead of volume *V*, we introduce density in this equation $\rho \propto 1/V$, then it will take the form $p\rho^{-\gamma} = const$. Differentiating it, we obtain:

$$\rho^{-\gamma} dp - \gamma p \rho^{\gamma-1} d\rho = 0, \text{ or } \rho dp - \gamma p d\rho = 0,$$

and for the adiabatic process:

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho}.$$

From the Mendeleev-Clapeyron equation it follows that:

$$p = \frac{\rho}{M} RT, \qquad (2)$$

where *R* is the universal gas constant, *M* is the molar mass, *T* is the temperature. Thus, the speed of sound can be determined as

$$\mathrm{v} = \sqrt{\gamma \frac{RT}{M}} \qquad (3)$$

The formula (3) is called the Laplace formula[1].

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[1] For the first time, formula (1) was used by Newton to calculate the speed of sound. Calculating the derivative *dp*/*dρ*, he proceeded from the assumption that compression and rarefaction in a sound wave occur at a constant temperature. From Eq. (1) and Eq. (2) at *T=const* it follows that $\mathrm{v} = \sqrt{\frac{RT}{M}}$. According to this formula, the speed of sound in air under normal conditions is v = 280 m/s, which significantly differs from the experiment (about 330 m/s). The reason for this discrepancy was precisely eliminated by Laplace, who showed that sound propagation is an adiabatic process.

In this practical, the method of superposition of oscillations is used to measure the speed of sound in a gas. The phone, powered by a sound generator, emits sound waves. A microphone, placed at a distance L from the phone, picks up these waves. The alternating voltage from the sound generator output is applied to the "X" input of the oscilloscope, and the alternating voltage from the microphone is applied to the "Y" input. Thus, the result of the superposition of two mutually perpendicular harmonic oscillations of the same frequency is an ellipse, and the phase difference between the oscillations is

$$\Delta\varphi = \frac{2\pi}{\lambda} L,$$

where λ is the wavelength. When the oscillation frequency f is gradually changed, the beam path on the screen also changes. When the phase difference is $\Delta\varphi = n\pi$ (n = 1, 2, 3 ...), the ellipse degenerates into a straight line that passes for even n values through the first and third quadrants of the screen, and for odd ones through the second and fourth quadrants. It is easy to show that the straight lines on the oscilloscope screen will be observed under the condition:

$$f = n\frac{\mathrm{v}}{2L},$$

where n= 1, 2, ... is an integer number. When a straight line transforms into an ellipse as the oscillation frequency changes, and then into a straight line again, the number n is changed by one. Consequently,

$f_n - f_k = (n-k)\frac{\mathrm{v}}{2L}$, where f_n and f_k are the frequencies corresponding to integers n and k. Thus, we can express the speed of sound as:

$$\mathrm{v} = \frac{2L(f_n - f_k)}{n-k}. \qquad (4)$$

DESCRIPTION OF THE EXPERIMENTAL SETUP

On the right you see a photo of the experimental setup. Its diagram is shown below in Fig. 1.

Inside the sealed tube C with air (it is suspended vertically on the wall) there are reinforced: sound source - telephone capsule T (telephone), connected to the output of the sound generator SG, and the microphone M. The frequency is determined using the scale of the sound generator. The signal from the microphone is fed to the "Y" input of the oscilloscope, on the "X'' input, the signal comes directly from the sound generator.

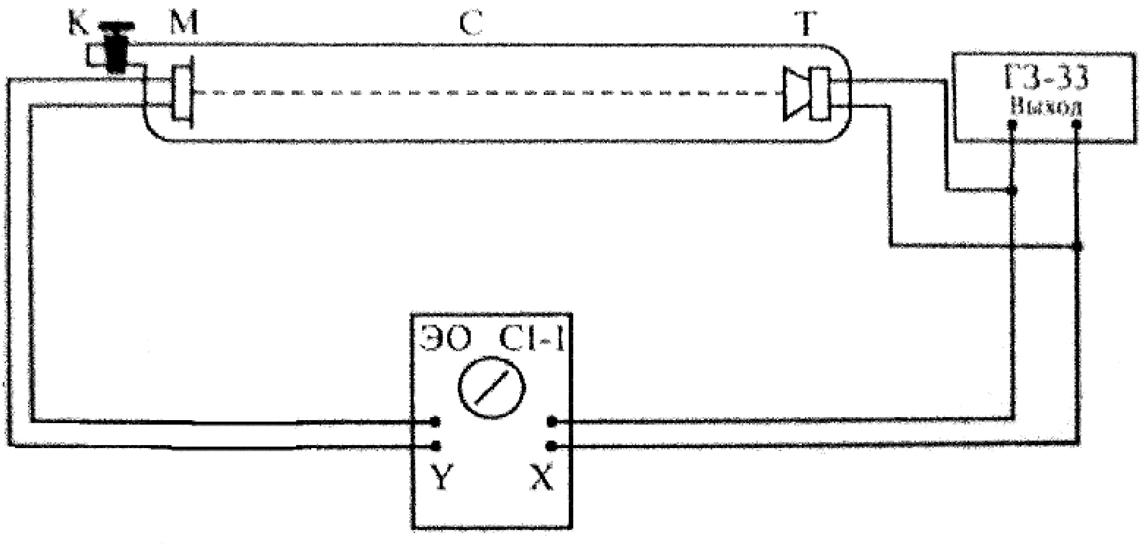

Fig. 1. Experimental setup

At the outlet of the glass tube *C*, there is a tap *K* that is used for connecting the tube to the pump or to a balloon with helium. The pumping pressure in the tube is controlled by a pressure gauge.

MEASUREMENT AND DATA PROCESSING

Task 1. Determination of the speed of sound in the air

Before starting, pump out the tube C by using the pump for 2 minutes, and then fill the tube with air by opening the tap K.

1.1 Increase gradually the frequency of the generator, starting from 1000 Hz (this restriction is imposed by the amplitude-frequency characteristics of the microphone and telephone). You need to obtain a series of consecutive values of frequency at which the straight lines in the first to third and second to fourth quadrants are observed on screen of the oscilloscope. Make sure that the results are reproducible if you take the measurements decreasing the frequency.

1.2 Plot the graph using the obtained results. Use the serial number of the measurement for the abscissa axis, and the frequency f for the ordinate axis. Through the points of the graph, draw the best fit line. From the slope of this line, using the formula (4), determine the average value of the speed of sound and estimate the error.

1.3. Calculate the adiabatic constant γ and estimate the error of the results. Compare the experimental value with the theoretical one for air.

Task 2. Determination of the speed of sound at a reduced pressure

Turn on the pump and, controlling the pressure in the tube with the gauge, evacuate the air down to a pressure of 0.4 - 0.5 atm. Measure the speed of sound at this pressure. Make a conclusion about the dependence of v on pressure.

Task 3. Determination of the speed of sound in helium

Completely pump air out of the tube and fill it with helium from the balloon. Measure the speed of sound in helium. Calculate γ. Estimate the accuracy of the results; compare the obtained values of v and γ with the theoretical ones.

Task 4. Determination of the speed of sound in a mixture of gases

Pump out helium down to a pressure of 0.6-0.7 atm. Turn off the pump and add some air to the tube, briefly opening tap K. Measure the speed of sound in an air - helium mixture. From the velocity value, calculate the ratio of the partial pressures of helium and air in the tube.

QUESTION AND EXERSIZES

1. Based on the first law of thermodynamics, obtain the Poisson equation for the adiabatic process.
2. Which gas will cool more strongly during adiabatic expansion from pressure p_1 to pressure p_2: helium or air?
3. Get the Laplace formula for the speed of sound in an ideal gas. Does sound velocity depend on pressure?
4. Calculate the speed of sound in dry air and in helium at a temperature of 20 °C.
5. Find the molar heat capacities C_p and C_V for a mixture containing: a) 2 moles of He and 1 mole of N_2; b) 1 g of He and 14 g of N_2.
6. Find the speed of sound in a mixture consisting of 1 mole of He and 1 mole of air.