PRACTICAL 4

STUDY OF THE GAS DIFFUSION COEFFICIENT DEPENDENCE ON THE GAS PRESSURE

Objective: experimental determination of the diffusion coefficient of water vapor in the air.

Equipment and accessories: instrument for determining the diffusion coefficient, ocular micrometer, lens, Komovsky pump, analog vacuum gauge, syringe with water, stopwatch.

INTRODUCTION

Diffusion is the phenomenon of the penetration of two contacting substances into each other. In this work, the diffusion coefficient of water (alcohol) vapor in the air is determined by evaporation of a drop of water (alcohol), assuming that the diffusion is stationary.

The mass of vapor dm, diffused through the surface dS during time dt is determined by the Fick equation:

$$dm = -D\frac{\partial \rho}{\partial r} \cdot dSdt\,, \tag{1}$$

where $\partial\rho/\partial r$ is the density gradient of saturated water vapor, D is the diffusion coefficient.

Under stationary conditions, the flux flow through a hemisphere of arbitrary radius r is constant and equals to

$$dm = -D\frac{\partial \rho}{\partial r} \cdot dSdt\,. \tag{2}$$

It follows that

$$r^2\frac{\partial \rho}{\partial r} = C_1 = const\,. \tag{3}$$

From (3) you can find the density gradient of the vapor:

$$\frac{\partial \rho}{\partial r} = \frac{C_1}{r^2}\,. \tag{4}$$

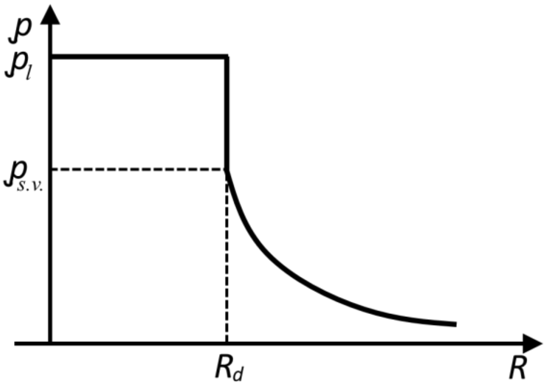

Fig. 1

Substituting (4) into the diffusion equation (1):

$$dm = -D \cdot dS \cdot dt \frac{C_1}{r^2}. \qquad (5)$$

To determine the constant C_1, the vapor density dependence on the distance r is used. Integrating (4), we obtain:

$$\rho(r) = \int \frac{C_1}{r^2} dr + C_2 = -\frac{C_1}{r} + C_2. \qquad (6)$$

The constant C_2 is found using the boundary conditions of the problem. Inside a drop, the density of water does not change with a change in value of r. At distances $r >> R$, where R is the droplet radius, the vapor density decreases according to equation (6). Dependence of density on the distance is shown in Fig. 1), and at $r \to \infty$ $\rho \to f\rho_{s.v.}$, where f is the relative humidity of the air, $\rho_{s.v.}$ – density of saturated vapors. Consequently $C_2 = f\rho_{s.v.}$. At the same time if $r = R$, vapor density $\rho = \rho_{s.v.}$, and $C_1 = -\rho_{s.v.}(1-f)R$, hence

$$\rho = \rho_{s.v.}(1-f)\frac{R}{r} + f\rho_{s.v.}; \qquad \frac{\partial \rho}{\partial r} = -\frac{\rho_{s.v.}(1-f)R}{r^2}. \qquad (7)$$

Substituting this value of the density gradient in (1), we get:

$$dm = D \cdot dS \cdot dt \frac{\rho_{s.v.}(1-f)R}{r^2}. \qquad (8)$$

If the conditions of the experiment are stationary, then the mass of vapors diffusing during time dt through a spherical surface with radius r is equal to:

$$dm = D \cdot 4\pi r^2 \frac{\rho_{s.v.}(1-f)R}{r^2} dt = D \cdot 2\pi\rho_{s.v.}(1-f)Rdt. \qquad (9)$$

At the same time, as the radius of the spherical drop decreases from R to (R-dR), the change in its mass dm_d is equal to:

$$dm_d = \rho_l 4\pi R^2 dR, \qquad (10)$$

where ρ_l – density of the liquid.

Considering that the decrease in the drop mass is equal to the mass of the diffused vapor: $dm_d = -dm$, using equations (9) and (10), we can relate the diffusion coefficient D to the rate with which the drop radius R changes:

$$D = -\frac{\rho_l}{\rho_{s.v.}(1-f)} \cdot R\frac{dR}{dt} = \frac{\rho_l}{2\rho_{s.v.}(1-f)} \cdot \left|\frac{d(R^2)}{dt}\right|. \qquad (11)$$

From the expression obtained, it is clear that for the experimental determination of D, it is necessary to measure the change in the drop radius with time under stationary conditions.

DESCRIPTION OF EXPERIMENTAL SETUP

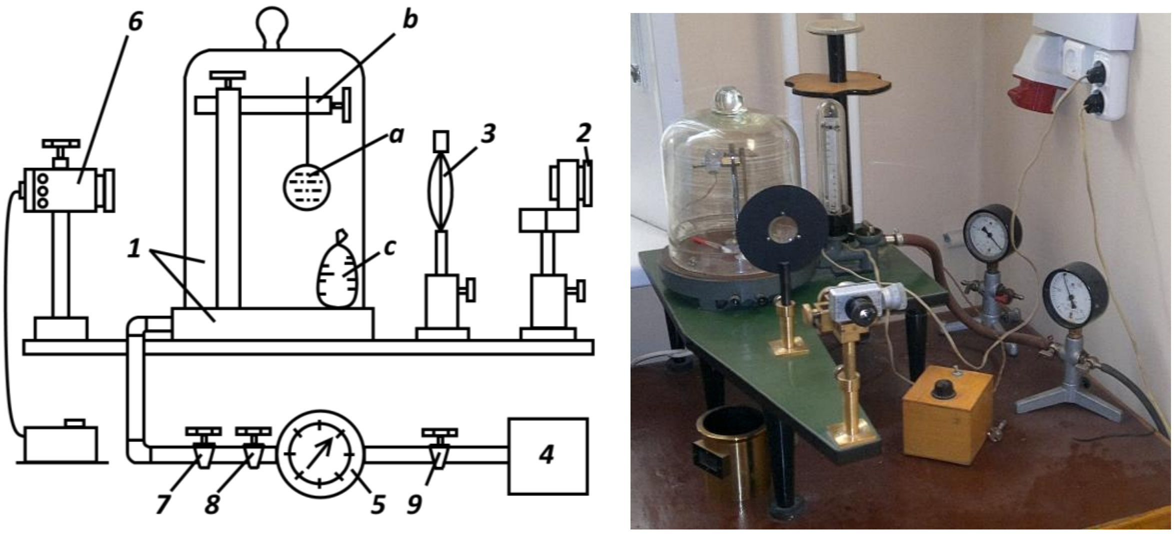

Fig. 2. Experimental setup

The experimental setup shown in Fig. 2. It consists of a glass-bell with a vacuum plate *1*, an ocular micrometer *2*, a lens *3*, a pump *4*, a vacuum gauge *5*, an illuminator 6, a three-way valve *7* and two-way valves *8* and *9*.

Under the glass-bell there are a drop of liquid *a* and the absorber *c*. The drop is placed in the ring attached to the holder *b*. The air pressure under the glass-bell can be changed using a valve 7 and a pump. An optical system consisting of a lens *3* and an ocular micrometer *2* serves to measure the diameter of the drop. The time is measured using a stopwatch.

MEASUREMENT AND PROCESSING RESULT

Preparing the setup

Remove the glass-bell. Using a syringe, hang the drop to the holder. Obtain a clear drop image in the eyepiece of the ocular micrometer.

Task 1. Measurement of the diameter of an evaporating drop at atmospheric pressure

Measure the distance from the drop holder to the lens with a ruler. Using the lens formula, calculate the distance from the lens to the actual image of the drop and the magnification given by the lens. The focal length of the lens F is 11 cm.

At atmospheric pressure, measure the droplet image diameter using the ocular micrometer and start the stopwatch. Measure the diameter at regular successive intervals. Make 5 measurements. At atmospheric pressure, measure the drop radius after 3-5 minutes, choosing the magnitude of the time interval depending on the rate of evaporation of the drop.

Task 2. Measurement of the diameter of an evaporating drop under reduced pressure

Suspend a new drop of water and close it with a bell, firmly rubbing it against the surface of the dish. Turn on the pump and pump out air down to 0.5 atm. Again, make 5 measurements of the radius of the evaporating droplet at intervals of 2–3 minutes.

Repeat the measurement for another drop and pressure of 0.25 atm.

Write down in the protocol the room temperature and relative air humidity f (it is measured by a hygrometer hanging on the wall). Using reference table 1 (p. 6.), determine density of the saturated steam $\rho_{s.v.}$ corresponding to the room temperature. Results of all measurements are organized in a table as shown below:

Table. Experimental results

<table>
<tr><th>№</th><th>d, μm</th><th>R, μm</th><th>R^2, m^2</th><th>t, s</th><td rowspan="6">Distance a = cm
Increase Γ =
Temperature: °C
Relative humidity f = %
$\rho_{s.v.}$ = kg/m^3</td></tr>
<tr><td colspan="5">$p = 1$ atm</td></tr>
<tr><td>1
2
…</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="5">$p = 0.5$ atm</td></tr>
<tr><td>1
2
…</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="5">……</td></tr>
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Task 3. Calculation of diffusion coefficients at different pressures

For each series of measurements, construct a graph, setting off the time t along the abscissa, and the square of the radius of the drop R^2 along the ordinate axis. According to the formula (11), the dependence $R^2(t)$ should be linear. Using the graph, determine the value of the derivative $\frac{d(R^2)}{dt}$ and its error. Calculate the diffusion coefficient using (11) at different pressures.

Task 4. Estimation of the free path of vapor molecules

Knowing the room temperature, calculate the average velocity of the water vapor molecules using the formula $\bar{v} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}}$, where R is the molar gas constant, M is the molar mass. From the formula $D = \frac{1}{3}\bar{v}\lambda$, find the length of the free path λ of the molecules of water vapor (alcohol vapor) in the air for the obtained values of the coefficient D.

For calculating, assume the liquid density $\rho_l = 10^3$ kg/m^3.

QUESTIONS AND EXERCISES

1. The length of the free path of the molecules depends on the concentration of the gas n and the scattering cross section $\sigma = \pi d^2$ (d - the effective diameter of the molecules): $\lambda = \frac{1}{\sqrt{2} n\sigma}$. Think about how to change this formula to estimate the free path of water molecules in the air.
2. How does the mean free path of molecules depend on pressure and temperature? At what pressure does the mean free path of air molecules equal to 1 mm, if at atmospheric pressure it is equal to $6*10^{-6}$ cm?
3. How does the gas diffusion coefficient depend on pressure and temperature?
4. Based on the differential equation (11), calculate how long the drop radius will be halved.
5. Using only dimensional considerations, determine the dependence of the average diffusion displacement of particles on time.
6. If the smell of an odorous substance spreads by diffusion over a distance of 1 m during the time t_1, then in what time t_2 will it spread to 10 meters?

Supplementary materials

1. Density ρ of saturated water vapor at various temperatures

Temperature, °C	$\rho_{s.v.}$, g/m^3	Temperature, °C	$\rho_{s.v.}$, g/m^3	Temperature, °C	$\rho_{s.v.}$, g/m^3
13	11.4	17	14.5	21	18.3
14	12.1	18	15.4	22	19.4
15	12.8	19	16.3	23	20.6
16	13.6	20	17.3	24	21.8

2. Diffusion coefficient D (at atmospheric pressure)

Diffusing substance	**Main component**	Temperature, T °C	Diffusion coefficient, m^2/sec
Water vapor	Air	0	$0{,}23 \cdot 10^{-4}$
Ethyl alcohol vapor	Air	0	$0{,}10 \cdot 10^{-4}$