PRACTICAL 3

DETERMINATION OF THE VISCOSITY COEFFICIENT OF AIR

1 Introduction

Real liquids and gases, are viscous. This phenomenon together with diffusion and thermal conductivity, belongs to the class of transport phenomena. If in the process of laminar flow, different layers of liquid or gas have different velocities, then with randomly moving molecules, these layers will exchange momentum. As a result, a viscous friction force F arises between the layers, while the magnitude of this force is determined by the Newton formula

$$F = \eta \frac{du}{dx} S, \qquad (1)$$

where η is the coefficient of viscosity (or internal friction) of the medium, u is the flow velocity of the liquid or gas, x is the coordinate perpendicular to the flow direction along which the velocity u changes, and S is the surface area of the layer. The speed of leveling is determined by the ratio $\frac{\eta}{\rho}$, where ρ is the density of the gas or liquid. The value $\nu = \frac{\eta}{\rho}$ is called the kinematic viscosity, and the value η is also called the dynamic viscosity.

Because of the viscosity of the laminar flow of air through the capillary, the flow velocities of infinitely thin cylindrical air layers located at different distances from the axis of the cylinder will be different.

The friction force acting on the elementary cylindrical volume and applied to the side surface of the cylinder, with the steady movement of air in the capillary, balances the difference in pressure forces acting on the cylinder base. In this case, the volume of air ΔV flowing through the cross section of a capillary of radius r over time t (with a pressure difference Δp at the ends of the capillary length l) is determined by the Poiseuille formula:

$$\Delta V = \frac{\pi r^4 \Delta p}{8 \eta l} t, \qquad (2)$$

Measuring the volume of air ΔV, that has flowed through the capillary during time t, one can find the viscosity:

$$\eta = \frac{\pi r^4 \Delta p\, t}{8 \Delta V l}. \qquad (3)$$

The Poiseuille formula (2) was derived under an assumption that air density was constant while the flow was laminar. In the experimental setup, which will be described below, the differential pressure is less than one percent of the atmospheric pressure, so the first assumption is well satisfied. As for the nature of the movement (laminar or turbulent), it is determined by the dimensionless Reynolds number:

$$\mathrm{Re} = \frac{\rho u r}{\eta}. \qquad (4)$$

In smooth round tubes, the transition from laminar to turbulent flow occurs at Re ~ 1000.

2 Experimental setup

A schematic of the setup, which is used to experimentally determine η, is shown in Fig. 1.

The main part of the device is a capillary *1*, through which air flows from the atmosphere to the gas meter *2* (it is shown horizontally in the figure while in the real setup it is fixed in a vertical position on the vessel of the gas meter, as shown in Fig. 1). When water flows out of the gasometer *2* at the ends of the capillary, a differential pressure Δp is created, measured by an inclined alcohol manometer *3*. The tube of the manometer is not installed vertically but at an angle $\alpha = 30^o$ to the horizon, which increases the sensitivity of the device. If during the measurement the alcohol column in the manometer shifted Δx, then the pressure difference at the ends of the capillary $\Delta p = p_c g \Delta x \sin\alpha$, where $p_c = 800$ kg/m^3 is the density of alcohol. The volume of the air that has flown through the capillary is equal to the volume of water ΔV that has spilled from the gas meter. This volume is measured by a beaker.

The values of the angle of inclination of the tube of the manometer, the radius of the capillary r and the length of the capillary l are indicated on the plate on the setup:

$$\alpha = 30^o,\ l = 36\ \text{cm},\ r = 0.53\ \text{mm}.$$

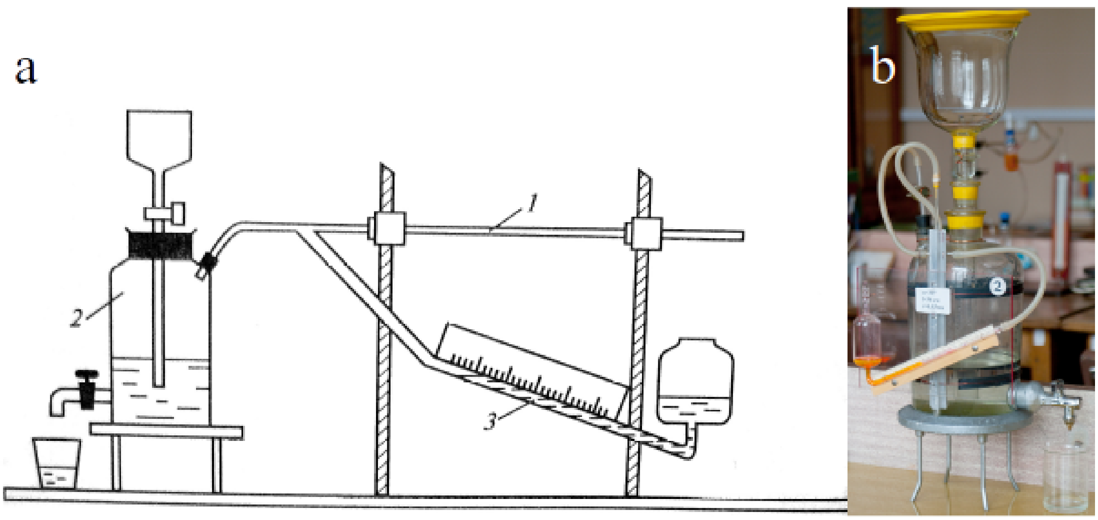

Fig. 1. a - A schematic of the setup; b – an actual image of the setup.

3 Measurement and data processing

Task 1. Determination of the viscosity coefficient of air.

Record the initial coordinate x_0 of the end of the alcohol column in the manometer. Adjust the flow rate of water from the gas meter using the tap. While working with the setup, make sure that the alcohol from the inclined tube of the manometer does not fall into the capillary and flexible hoses. Make the alcohol column displace along the scale of the manometer at a distance Δx of *20* to *80* mm and set at a certain point. Measure how much time t is needed for a volume of air $\Delta V = 100$ ml to flow through the capillary at the steady flow rate of air through the capillary (if during the measurement the alcohol column begins to move strongly, the measurement can be stopped with a smaller amount of water that has leaked, but ΔV should not be less than *50* ml). Repeat the measurements described above at five different values of the differential pressure Δp. Put down the results of measurements and calculation into table 1.

№	ΔV, ml	t, s	Δx, mm	Δp, Pa	$\Delta V/t$, m^3/s	η, Pa · s
1						
2						
3						
4						
5						

Plot the graph of Δp as the function of air flow $\frac{\Delta V}{t}$. Draw a line best fit through the origin (if any point is far from the straight line, it is considered to be an outlier). From the slope of this straight line, one can calculate the average value of η.

Measuring the average slope of the graph $\frac{\Delta p}{(\Delta V/t)}$, using the formula (3), calculate the coefficient of internal air friction η. Estimate the measurement error using the graph.

Record the ambient air temperature and atmospheric pressure (a thermometer and a barometer can be found in the laboratory) in the room. Calculate the density of air: $\rho = \frac{pM}{RT}$ ($M = 29$ g/mol - the molar mass of air, T – the absolute temperature). Calculate the kinematic viscosity of air $\nu = \frac{\eta}{\rho}$.

Using formula (4) estimate the value of the Reynolds number at the maximum value of Δp. To do this, calculate the average air flow rate u using known values of ΔV, r and t:

$$\Delta V = \pi r^3 u t, \qquad u = \frac{\Delta V}{\pi r^2 t}.$$

Make a conclusion about the nature of the flow of gas.

4 Questions

1. What physical quantities determine the coefficient of internal friction in gases? Consider also the case of technical vacuum.

2. How will the coefficient of internal friction of a gas change with a 2-fold increase of pressure (at T = const); 2-fold increase of temperature (at p = const)?

3. How will the coefficient of viscosity change with a 2-fold increase of pressure (at T = const)?

4. According to the value of η found in the handbook, estimate the free path of nitrogen molecules under normal conditions.

5. Derive the Poiseuille formula (2).

6. What law defines the flow rates of individual gas layers change from the wall to the tube axis under the condition of a laminar flow?

7. What is the theoretical relationship between the viscosity and the thermal conductivity of a gas? Check how well this ratio is fulfilled using the values of viscosity and thermal conductivity from the handbook for different gases (two or three).

8. The thermal conductivity of helium is 8.7 times greater than that of argon (under normal conditions). Find the ratio of the effective diameters of the atoms of He and Ar and compare the obtained numbers with data from the handbook.