PRACTICAL 2

STUDY OF DISTRIBUTION OF BROWNIAN PARTICLES IN THE HOMOGENEOUS FIELD OF GRAVITY

1 Introduction

Under conditions of thermal equilibrium in the absence of external fields, the probability of finding the ideal gas molecule in the vicinity of any point in the volume occupied by the gas is the same throughout the volume. That is why the concentration of particles under these conditions is the same in all parts of the volume.

However, if the gas is in an external field with potential energy *U(r)*, then the probability of finding a particle in unit volume in the vicinity of a point *r* is determined by the Boltzmann factor $\exp(-\frac{U}{kT})$ (where *k* is the Boltzmann constant, *T* is the absolute temperature). At the same time, the concentration of particles is proportional to the probability of their being in a certain volume region. Therefore, the average concentrations of particles n_1 and n_2, corresponding to the values of the potential energy U_1 and U_2, are related by the equation

$$n_2 = n_1 \exp(-\frac{U_2 - U_1}{kT}). \qquad (1)$$

The equation (1) expresses the so-called Boltzmann distribution. It is valid not only for an ideal gas molecules, but also for the aggregate of any other classical particles that are in an external field in a state of thermodynamic equilibrium. For example, Brownian particles, that are in equilibrium with gas or liquid molecules, obey the Boltzmann distribution.

The potential energy difference of $U_2 - U_1$ between two levels for Brownian particles is determined by the work A_{12} of the force acting on the particle as it moves from one level to another. In particular, for a Brownian particle in a liquid within a uniform field of gravity

$$U_2 - U_1 = A_{12} = (\rho - \rho_0) g V \cdot \Delta h, \qquad (2)$$

where ρ and ρ_0 are the densities of the Brownian particle and the liquid, respectively, V is the volume of the Brownian particle, g is the acceleration of free fall, Δh is the distance between the levels along the vertical.

From (1) and (2) one can get an expression that determines the volume of a Brownian particle:

$$V = \frac{kT \ln \frac{n_1}{n_2}}{(\rho - \rho_0) g \cdot \Delta h}, \qquad (3)$$

where n_1 and n_2 are average concentrations of particles in the lower and upper layers of the liquid.

In this practical, the object of the study are particles of finely ground watercolor paints suspended in water. These particles are in a state of thermodynamic equilibrium with water molecules, and, therefore, all the reasoning given above is applicable to them. To calculate the volume of these particles using the relation (3), it is necessary to determine the concentrations of particles at two levels, the distance between which is a certain value Δh. The studied Brownian particles are observed through a microscope.

2 Experimental setup

The aqueous solution of watercolor paint is poured into the recess of the slide and is covered with a cover glass on top so that there are no air bubbles under them. Then the slide is mounted on the microscope stage.

Because of the shallow depth of the image field of the microscope, it is possible to see simultaneously paint particles in a very thin horizontal layer of liquid in the field of view of the microscope. By moving the microscope tube by a small distance Δh_d, another layer of liquid, which is separated from the first by a distance $\Delta h = n_w \Delta h_d$, where $n_w = 1.33$ is the refractive index of water, can be observed. The shift of the tube Δh_m is measured with the micrometer screw. A diaphragm is placed in the eyepiece of the microscope to limit its field of view.

In order to determine the ratio of average concentrations of Brownian particles n_1/n_2 at two levels, at each level at least 30 counts of the number of particles N_i, simultaneously observed in the field of view, limited by an ocular diaphragm, should be made. The average number of particles N observed in a given layer is proportional to the concentration of particles at the level of this layer. Therefore, the ratio of particle concentrations at different heights can be taken as equal to the ratio of the average numbers of particles N_1 and N_2 at the corresponding levels.

3 Measurement and data processing

Task 1. Observation of Brownian motion and determination of average numbers of particles $\bar{N}_1$ and $\bar{N}_2$ at two levels

Begin an observation by lowering the objective close to the slide. Rotating the micrometer screw towards yourself, slowly raise the microscope tube until particles appear in the field of view. Make sure that with small movements of the tube, the images of some particles disappear, while others appear.

Insert the diaphragm in the microscope eyepiece to limit the field of view. Choose a lower level so that 5-7 particles are observed simultaneously in a limited field of view.

Count the number of particles simultaneously observed in the field of view. Repeat the observation and counting 30 times at regular intervals (for example, 10 sec after each observation). The arithmetic mean of the number of particles will determine the desired value of $\bar{N}_1$ (the so-called “sample mean”).

To determine the average number of particles $\bar{N}_2$ at another level, lift the microscope tube by 30–50 μm and repeat similar measurements.

Calculate for each level the average number of particles $\bar{N}$, the RMS error s and the RMS error of the sample mean σ'. Recall what characterizes each of these errors. What is the expected difference between the true average value of a random variable and the sample average?

Task 2. Determination of particle volume

Determine the volume of paint particles according to the formula (3), noting that the density of the paint is $\rho = 1.06 \times 10^3$ kg/m^3. (Take the temperature of the liquid to be equal to the room temperature.) Estimate the error of the average volume of Brownian particles.

4 Questions

1. What explains the random movement of particles suspended in a liquid or in a gas (the Brownian motion)? How does the nature of the motion of Brownian particles depend on their sizes?

2. Estimate the average speed of Brownian particles (calculate the mass of particles from the volume measured in the practical).

3. Write the expression and plot the concentration *n(h)* of Brownian particles as a function of height *h* for different temperatures. What is the physical meaning of the area under these curves?

4. Using the results of your measurements, calculate at what height the average concentration of Brownian particles decreases by a factor of 2.

5. Calculate an average volume of a single water molecule, and compare it with the volume of the Brownian particle obtained in the practical.

6. Is it possible, by increasing the number of samples, to reduce the mean square error of a random variable? Is it possible, by increasing the number of samples, to determine the average value of a random variable arbitrarily accurately? Why is it recommended to take a large number of samples of particles visible in the field of view of the microscope?

7. Using the results of your experiment, construct histograms for each of the levels: plot the number of N_i Brownian particles along the x-axis, simultaneously observed in the field of view; plot the number of case with the given number of observed Brownian particles along the y-axis. Mark the following points $N, N \pm s, N \pm 2s$ on the x-axis. What is the proportion of cases in the interval of $\bar{N} \pm s$ and $\bar{N} \pm 2s$? Compare the obtained results with the predictions of the theory of probability.