PRACTICAL 1.17

DETERMINATION OF YOUNG'S MODULUS

Objective: The experimental determination of the Young's modulus by using he wire elongation method.

Equipment: special setup, a set of weights, indicator (distance amplifying instrument)

INTRODUCTION

The tensile stress that is produced within the deformed body is proportional to its fractional extension (or strain). In the case of linear longitudinal deformation of the cylindrical sample, this ratio (Hooke's law) is usually written in the following form:

$$\sigma = E\varepsilon, \qquad (1)$$

where $\sigma = F/S$ is stress, $\varepsilon = \Delta l/l$ is a fractional change in length, E is the modulus of elasticity (Young's modulus), F is the deformation force, l and S are length and cross section of a sample.

If one will measure the deformation force and the corresponding elongation of the sample Δl, the Young's modulus can be determined by using the expression:

$$E = \sigma/\varepsilon = Fl/(S\Delta l). \qquad (2)$$

EXPERIMENTAL SETUP

The experimental setup is presented on the Figure 17.1. The studied sample K (wire) is fixed to the upper bracket N. The suspended platform *D* with a set of weights P is also fixed to the same bracket. The weights are shifted to the platform *C* causing to the deformation of the tested wire. In this way, the upper bracket is always under a constant stress, and the deformation of the bracket does not introduce the errors for measurement of the elongation of the tested wire. The measurement of the elongation of the wire is carried out by using an indicator *B* that is fixed to the bottom bracket *A*.

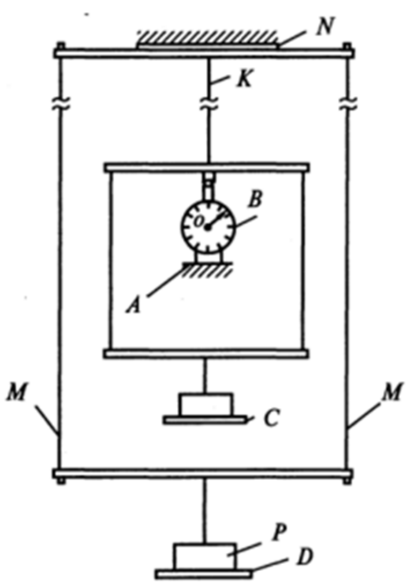

Figure 17.1. Experimental setup.

MEASUREMENT AND DATA PROCESSING

The tested wire is loaded by a 1 kg-weight, and then the indicator is set at zero level by rotating the outer ring of the indicator display. Then, the weights (0.5 kg of each) are shifted on the platform C alternately, and one fixes the corresponding elongation of wire. The obtained data is plotted in the coordinates [σ,ε], and then the Young's modulus can be calculated. Length and diameter of the wire are given in the description of experimental setup.

Experimental task. Carry out the measurement of the wire elongation for four values of the deformation force. The recommended values of the deformation force are 5, 10, 15 and 20 N. Repeat the measurements three times for each value of the deformation force. Plot the obtained experimental data: the tensile stress σ as a function of the fractional change in the length ε.

Calculate the Young's modulus for the wire. Use the look-up table for determination of the wire material. Evaluate the errors of the experimental results. Put all experimental data and calculations of F, Δl, σ,ε, E in a Table.

QUESTIONS AND EXERCISES

1. What type of deformation is named elastic?
2. Describe and explain the method and experimental setup of the laboratory work.
3. How is the volume of a sample changed under longitudinal elongation or compression?
4. What type of errors (random or systematic) determines the accuracy of your measurements?
5. How are deformation (strain) distributed in the cylindrical body if the body is at rest? If it moves with an acceleration?
6. How is deformation (compressive strain) distributed in a homogeneous cylindrical body that is caused by the force of gravity (gravity parallel to the cylinder axis), if: a) it is suspended from the upper end to the fixed support; b) it stands on a fixed support?

PRACTICAL 1.18

DETERMINATION OF SHEAR MODULUS

Objective: The experimental determination of the shear modulus of the cylindrical wire by the method of torsion oscillations.

Equipment: torsion pendulum, micrometer, ruler, stopwatch.

INTRODUCTION

Screwing of wire causes to the shear deformation inside it. At small angles of rotation φ, such deformations are elastic, so the torque is proportional to the angle of screwing of the wire:

$$M = -k\varphi . \qquad (1)$$

The coefficient k in the Eq.(1) depends on the elastic properties of the material of the wire, which is called the shear modulus G (or modulus of rigidity). According to Hooke's law one can find the relationship between k and G [15]:

$$k = G\frac{\pi r^4}{2l}, \qquad (2)$$

where *r* is the radius of the wire, *l* is the length of the wire. With regard to (2), the Eq. (1) takes the form:

$$M = -G\frac{\pi r^4}{2l}\varphi . \qquad (3)$$

This ratio indicates the two possible methods of experimental determination of the shear modulus:

- Static method. The measurement of the torque of external forces (numerically equal to the moment of the elastic forces about the rotation axis) and the corresponding angle of twist of the wire;
- Dynamic method. The measurement of the period T of torsional oscillations when the cylindrical body is suspended at the free end of the wire.

If I is the moment of inertia of the pendulum about the rotation axis, the period of oscillations and the shear modulus are given by

$$T = 2\pi \cdot \sqrt{\frac{I}{G\frac{\pi r^4}{2l}}} \quad \text{and} \quad G = \frac{8\pi l}{r^4} \cdot \frac{I}{T^2} . \qquad (4)$$

The dynamic method does not require precise instruments for measuring angles and torque moment, and it is widely used for the experimental determination of the shear modulus. This method is used in the current work.

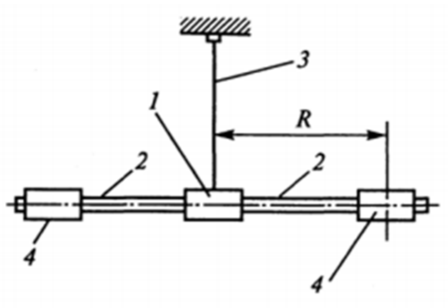

Figure 18.1. Experimental setup.

EXPERIMENTAL SETUP

The experimental setup is presented on the Figure 18.1. The torsion pendulum consists from a rod *2* that is suspended on a wire *3*. The experimental task is determined the shear modulus of the material of the suspended wire. Two identical cylinders *4* are loaded on the rod. Moving the cylinders along the rod allows to change the moment of inertia of the pendulum.

To determine the shear modulus, one needs to measure the length and the radius of the studied wire, the period of oscillations and the moment of inertia of pendulum.

The first three values can be measured directly. The moment of inertia is defined in the following way. One can measure the period of oscillations of the pendulum for two different positions of the cylinders on the rod. It gives the determination of relationship I/T^2 for the pendulum:

$$\frac{I}{T^2} = \frac{I_1 - I_2}{T_1^2 - T_2^2} = \frac{2m\left(R_1^2 - R_2^2\right)}{T_1^2 - T_2^2}, \qquad (5)$$

where R_i is a distance between the center of mass of one of the cylinders and the rotation axis. Thus, the Eq. (4) for the shear modulus takes the form:

$$G = \frac{16\pi \cdot m\left(R_1^2 - R_2^2\right)l}{\left(T_1^2 - T_2^2\right)r^4}. \qquad (6)$$

All quantities on the right-hand side of Eq.(6) can be measured directly.

MEASUREMENT AND DATA PROCESSING

Experimental task. Measure the time of 50 oscillations of the torsion pendulum at two different positions of the cylinders and calculate the shear modulus for the wire.

Use the look-up table for physical quantities for determination the material of the wire. Add all measured (m, R_1, R_2, t_1, t_2, l, r) and calculated (T_1, T_2, G) data in the Table. Evaluate the errors for the determination of shear modulus G by the given method.

QUESTIONS AND EXERCISES

1. How is the shear strain distributed along the wire in static and dynamic cases?
2. What measurement contributes to the maximum error in the determination of the shear modulus? Do systematic or random errors determine the accuracy of the experiment?
3. What position of the two cylinders on the rode causes to the minimal experimental error?
4. What kind of deformation is called elastic?
5. What factors limit the amplitude of the harmonic oscillations of the torsion pendulum?
6. Prove the Eq. (3) and Eq. (5).