PRACTICAL 1.4

SUPERPOSITION OF THE HARMONIC OSCILLATIONS

Objective: Studying the motion trajectory and the time-coordinate dependence of a point mass which takes part in two different oscillations, which are either co-directional or perpendicular ones. Studying oscillations of the electrical quantities with the oscilloscope.

Equipment: sand pendulum, stopwatch, ruler, paper, sand, oscilloscope, two sound generators, transformers, an electrical circuit.

INTRODUCTION

In the work, the trajectory of motion and the time-coordinate dependencies of a point mass (in other words, *material point*), which is involved in several oscillating processes, are studied. The motions take place either along a single direction or in mutually perpendicular directions.

1. Superposition of harmonic oscillations that coincide in frequency and direction.

When two harmonic oscillations of different amplitudes and initial phases $x_1=a_1\cos(\omega t)$ and $x_2=a_2\cos(\omega t+\varphi)$ are superimposed, the motion of a point mass is well described by equation:

$$x=x_1+x_2= a_1\cos(\omega t) + a_2\cos(\omega t+\varphi)= a\cos(\omega t+\varphi_0) \qquad (1),$$

with $a\sqrt{a_1^2+a_2^2+2a_1a_2\cos\phi}$ and $\mathrm{tg}\phi_0=\frac{a_2\sin\phi}{a_1+a_2\cos\phi}$.

2. Superposition of mutually perpendicular harmonic oscillations.

Equation (2) describes the trajectory of motion of a point mass, which is involved in two oscillation processes of same frequency ω ($x=x_m\cos(\omega t)$, $y=y_m\cos(\omega t+\varphi)$), and taking place along the mutually perpendicular axes. In this case, the point mass makes an elliptical motion, with the axes of the ellipse differ from the reference axes (e.g., Descartes coordinates) in general (Fig. 4.1).

$$\frac{x^2}{x_m^2}+\frac{y^2}{y_m^2}-\frac{2xy}{x_m y_m}\cos\phi=\sin^2\phi \qquad (2)$$

As can be seen from the figure, the trajectory of the point mass lies inside the rectangle whose sides are parallel to the Descartes axes and equal to 2x and 2y, and the center coincides with the origin (the equilibrium position of the oscillating point); the phase difference between the superimposed oscillations is determined by the formula $\sin\varphi=x_0/x_m$, where x_0 - the x coordinate value for y = 0. In case when the frequency ratio of the superimposed oscillations is an integer, the motion trajectories are closed lines, which are referred to as the Lissajous figures. The shape of these figures depends on the ratios ω_1/ω_2, x_m/y_m and the initial phase difference φ.

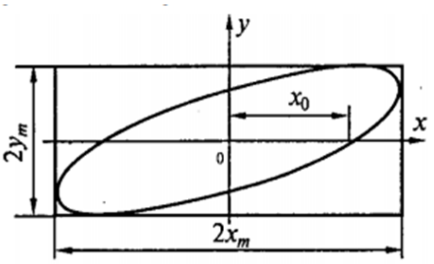

Fig. 4.1. Motion trajectory.

Also, an oscilloscope and electronic circuit are used for observation of the superimposed oscillations, for that the motion of the electron beam on the screen of the oscilloscope under the influence of an alternating electric field is observed. Trajectory of motion of a point mass is studied with use of the sand pendulum.

DESCRIPTION OF THE EXPERIMENTAL SETUP

1. The resultant trajectory of a point mass subjected to two oscillation processes is visualized with use of a sand pendulum, which is a massive body with a funnel for sand. The funnel is suspended on two threads to the frame (Fig. 4.2.). In some approximation, such pendulum can be considered as a mathematical pendulum, which is capable for oscillating in a perpendicular direction to the plane in which the suspension threads are located in the state of equilibrium. Using the screw K, the length of the threads can be adjusted.

The threads are connected by a sleeve in the point C. The sleeve can be moved in height, thus changing the length ratio of the threads. The pendulum funnel is filled with sand, which can pour through a narrow opening in the bottom of the funnel onto the paper under the pendulum. If the pendulum swings in two mutually perpendicular directions, the poured sand will form a curve on the sheet of paper, which would represent the trajectory of the pendulum motion. By varying the length ratio of the threads, one can observe different Lissajous figures. Lissajous figures can also be observed on the oscilloscope screen, if two harmonic electrical signals are applied to the horizontal and vertical input of the oscilloscope from the various sound frequency generators, respectively.

2. Figure 4.3 shows an electrical scheme of the setup, which is used to study superposition of the electrical oscillations. Voltage is supplied through a step-down transformer to the A-C electrical circuit clips containing the resistance and reactance. The potential difference between points AB, BC, AC changes harmonically: $U_{AB}=U_1\cos(\omega t)$, $U_{BC}=U_2\cos(\omega t+\varphi)$, $U_{AC}= U_1\cos(\omega t)+ U_2\cos(\omega t+\varphi)$. The phase difference occurs due to the capacitance in the BC circuit branch. If the vertical input of the oscilloscope (Y input) is connected to the circuit sections AB, BC and AC, then the vertical displacement of the beam on the oscilloscope screen will be proportional to the potential difference, U_{AB}, U_{BC}, $U_{AC} = U_{AB} + U_{BC}$, respectively, and the corresponding sine wave will be observed on the screen. The offset between the sine waves along the horizontal X-axis will correspond to a phase shift between them. This phase shift can be calculated from the fact that the phase shift over one period is 2π radians.

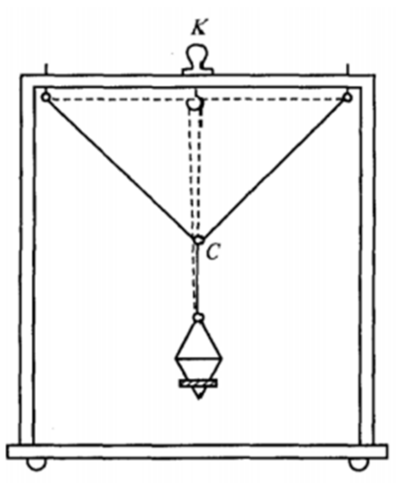

Fig. 4.2. Sand pendulum.

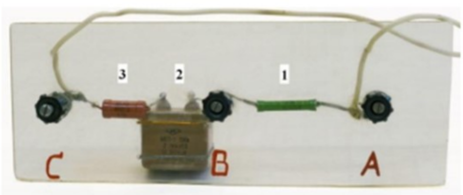

Fig. 4.3. Electrical circuit.

MEASUREMENTS AND DATA PROCESSING

Task 1. Observation of the motion trajectory of a sand pendulum.

By securing the sleeve on the frame hook, equalize the periods of the superimposed oscillations. Using a stopwatch, determine the period of oscillations T of the pendulum. Put a sheet of paper under the frame, mark a stationary pendulum projection and draw the coordinate axes through this point (in the frame plane and perpendicular to it). Take the pendulum in a random direction and release it, provide it with a little extra boost in the x or y direction. Do not lift the funnel above the table higher than 2 - 3 cm.

Get the trajectory of the resulting motion on the sheet of a paper by moving the latter along the table. Draw out the resulting curve with a pencil, determine the amplitude, phase shift, and write the equations for oscillations in each direction. Measured and calculated data for T $x_{m,}$, y_m, φ write down into a table.

Calculate the ratio of the lengths of the pendulums for which the ratio of the periods will be equal to 2. Place the sleeve on the threads to a position which satisfies to the found condition. Check the choice of the sleeve position by a direct measurement of the oscillations periods in the plane of the frame T_1 and in a direction, which is perpendicular to the plane of the frame, T_2. Measured and calculated data for l_1, l_2, T_1, T_2 write down into the table.

Deflect the pendulum aside; make it to oscillate by pushing it in the direction along the x-axis, and in the direction along the y-axis. Draw the curves formed by sand on a sheet of paper.

Task 2. Superposition of harmonic oscillations that coincide in frequency and direction.

Connect the Y-input of the oscilloscope with the AB section of the circuit (point A is connected to the terminal "Ground" of the oscilloscope). Using knobs "Frequency Range", "Frequency smoothly," "Amplitude synchronization" set a stable regime, which corresponds to one period of the oscillations, on the oscilloscope screen. The "synchronization" knob should be set to "Line". Choose the position of the "Attenuation" knob so that the amplitude of the displacement of the electron beam on the oscilloscope does not exceed 3 cm. For all further measurements the position of all knobs on the panel must remain intact. Draw the resulting curve on the tracing paper, noting the position of the horizontal and vertical axes on the grid of the oscilloscope screen.

Further, connect the Y-input of the oscilloscope with the circuit section (the terminal B is connected to the terminal "Ground" of the oscilloscope). Sketch the second curve on the tracing paper without moving the position of the horizontal and vertical axes relative to the coordinate grid of the oscilloscope. Calculate the phase shift of the second curve relative to the first one, from the displacement of the maximum of the second curve along the horizontal axis. Measure the amplitudes of both oscillations. Construct a vector diagram and find the amplitude and the initial phase of the resulting oscillations. To verify this result, connect the Y-input of the oscilloscope with the AC section of the circuit (the terminal A is connected to the terminal "Ground"). Sketch the third sine wave on the tracing paper. Measure the amplitude of the resulting oscillation and calculate its phase shift relative to the first one. Compare the values with those calculated from the vector diagram. Write down the measured and calculated data for amplitude and phase in the table.

Task 3. Superposition of the two mutually perpendicular oscillations with an oscilloscope.

Turn on the sweep generator of the oscilloscope. Apply voltage from the output terminals of the sound generator 1 at frequency ~ 200 Hz to the horizontal deflection plate (input *x*). Apply voltage from the output terminals of the sound generator 2 to the vertical deflection plates (input *y*). Get the curves on the screen of the oscilloscope, which are equal to the frequency ratio (1:1, 1:2, 2:1, 1:3 and 2:3) of the superimposed oscillations, by smoothly changing the output voltage frequency of the sound generator 2. Draw the observed curves.

QUESTIONS AND EXERCISES

1. Derive formula (1) and (2) using the method of vector diagrams and direct calculations.

2. Is it possible to set the knob "Synchronization" on the oscilloscope panel in the position "Internal", while working on the task 1?

3. Derive the equation of the trajectory of motion of a mathematical point (3), which vibrates in two orthogonal directions, if $\omega_1/\omega_2=1$ and the phase shift φ. Consider special cases $\varphi=0$, $\varphi=\pi/2$.

4. Using the results from the task 2, determine the kinematic parameters of the funnel motion along the elliptical trajectory: speed, normal and tangential acceleration, total acceleration, the radius of curvature at different points of the trajectory. Put down the equation of the trajectory in the parametric form.

5. How can one derive the ratio of frequencies of the adding vibrations from the shape of Lissajous curves?

6. Derive the equation of the trajectory of the point, which oscillates in two orthogonal directions, if $\omega_1/\omega_2=1$ and $\varphi=0$.