PRACTICAL 1.5

MEASURING MOMENT OF INERTIA OF A BICYCLE WHEEL

Objective: An experimental determination of the moment of inertia of a bicycle wheel by rotation and oscillation methods.

Equipment: a bicycle wheel with a pulley and sockets, a set of weights, a metal ball, an electric stopwatch, a millimeter ruler, a caliper.

INTRODUCTION

Two methods are used in this work to determine the moment of inertia of the body (a bicycle wheel): the method of rotation and oscillation method.

In the first case, the mass center of the body lies on the axis of rotation. The wheel is rotated by the tension force of a thread. The load m is suspended to the free end of the thread. The moment of inertia of the wheel I_w can be calculated from the dynamics equation of the rotational motion of the body relative to the fixed axis Z: $I_w\varepsilon=M_z$, if the angular acceleration ε and the moment of tension force M_z of the thread are known. These values can be determined by measuring the path h, travelled by the load m, during time t.

$$I_w = \frac{M_z}{\varepsilon} \mathrm{mR}^2 \left(\frac{\mathrm{gt}^2}{2h} - 1\right), \qquad \varepsilon = \frac{2h}{\mathrm{Rt}^2} \qquad (1)$$

Equation (1) is derived without consideration of the frictional forces. In order to take into account the frictional force of the wheel, which arises from its rotation, the maximum weight of the load Δm, which could be suspended on the thread with the wheel being in the standstill condition (i.e., the load of Δm is the load which just starts to drive the wheel into rotation), should be measured. Since the equilibrium condition is $\Delta mgR=M_{fr}$, the equation (1), with consideration of the frictional force on the axis of the wheel takes the following form:

$$I_w = (m - \Delta m)R^2 \left(\frac{\mathrm{gt}^2}{2h} - \frac{m}{m-\Delta m}\right) \qquad (2)$$

In the second case, the position of the mass center of body should be shifted below the axis of rotation. In this case, the wheel acts as a physical pendulum. If disturbed from its equilibrium position by a small angle, the pendulum starts to oscillate with a period that depends on its moment of inertia relative to the fixed axis:

$$T = 2\pi \sqrt{\frac{I}{m_p \mathrm{gd}}} \qquad (3)$$

where m_p - the mass of the pendulum and d - the distance from the center of mass to the axis of rotation. Equation (3) allows calculating the moment of inertia of the body from the measurement period of oscillation of the physical pendulum.

DESCRIPTION OF THE EXPERIMENTAL SETUP

The experimental setup is a bicycle wheel, which can rotate around a horizontal axis. The thread is wound onto the wheel pulley. The cup, where the cargoes are loaded, is attached to the free end of the thread. The movement time of the load is measured with an electric stopwatch. The distance h, which is travelled by the load m is determined by the meter ruler.

There are two identical small slots on the inner side of the wheel rim. The slots are located symmetrically along the diameter of the wheel. We get a physical pendulum capable of swinging around the wheel axis by placing a metal ball with weight m_0, into one of the slots.

In this case, the moment of inertia of the pendulum *I* is composed from the wheel moment of inertia and I_w and the moment of inertia of the ball I_b relative to the axis passing through the center of the wheel:

$$I_b = 0.4 m_0 r^2 + m_0 l^2$$

where *r* – the radius of the ball, *l* – the distance between the mass center or the ball and the axis of rotation. In this case equation (3) easily transforms into:

$$T = 2\pi\sqrt{\frac{I_b + I_w}{m_0 \mathrm{gl}}} \qquad (4)$$

The period T of the pendulum oscillation is measured (using an electric stopwatch) to determine I_w, ball mass m_0, the ball radius *r* and the distance *l*.

MEASUREMENTS AND DATA PROCESSING

Task 1. Determination of the moment of inertia of the wheel by the rotation method

Make the necessary measurements and calculate the moment of inertia of the wheel for three different loads. The time intervals *t*, needed for the cargo *m* to travel the distance *h*, are determined from three measurements in each case. Measured data for *t, h, m*, Δm, *R* and calculated data for I_w write down in the table.

Task 2. Determination of the moment of inertia of the wheel by the oscillation method

The oscillation period of the pendulum is determined from the total time of 10 completed motions. Measured data for m_0, *t, d, l* and calculated data for *T*, I_b, I_w write down in the table. Compare the values of I_w, obtained by the two methods.

Estimate the errors of I_w evaluation by the oscillation method and by the rotation method.

QUESTIONS AND EXERCISES

1. Derive the equations (1) - (4).
2. What is the necessary accuracy level for the values of π, *g* taken as the "table values", for I_w evaluation from the oscillation method and the rotation method?
3. Which of the two used methods (rotation or oscillation) gives a smaller error in the I_w evaluation?
4. What type of an error (random or systematic) determines the accuracy of the measurement of I_w in experiments you have carried out?
5. Determine the load speed and angular rotation speed at a time *t* after the start of the movement. Get the answer on the basis of the laws of dynamics and the law of conservation of the energy.
6. Derive the equation for the moment of inertia of the wheel.

PRACTICAL 1.6

STUDYING THE ROTATIONAL MOTION OF A SOLID

Objective: an experimental study of the rotational motion of a solid using the Oberbeck pendulum.

Equipment: the Oberbeck pendulum, an electric stopwatch, a caliper, a set of weights, ruler.

INTRODUCTION

The rotational motion of a solid body, relative to the fixed Z-axis, is described by the equation:

$$I_z\varepsilon = M_z, \qquad (1)$$

where I_z – the moment of inertia of a solid relative to a fixed axis, M_z - the projection of the moment of the external forces on the same axis, ε - angular acceleration.

The experimental verification of the equation (1) can be carried out by examining the relationship $\varepsilon(M)$ at a constant I or dependence $\varepsilon(I)$ at a constant M.

If the cylinder of the radius R rotates by the force of the tension of the thread, which is wounded on it, with a cargo dipping in a gravitational field, the value of M_z can be calculated from the measurements of the path h, travelled by cargo m over the time t: $M = m(g - a)R;$ $a = 2h/t^2$, *then*

$$M_z = m\left(g - \frac{2h}{t^2}\right)R,\ \varepsilon = \frac{2h}{\mathrm{R}t^2}. \qquad (2)$$

The equation (2) is derived without taking into account the frictional forces on the axis of the instrument. Taking into account the frictional forces, the equation (2) can be written as follows (see the work 1.5.):

$$M_z = (m - \Delta m)R\left(g - \frac{2h}{t^2}\cdot\frac{m}{m-\Delta m}\right) \qquad (3)$$

where Δm – the mass of the cargo, which is sufficient to start the motion of the system. Experimental verification of the fundamental equation of dynamics of rotational motion of a solid can be implemented with the Oberbeck pendulum.

DESCRIPTION OF THE EXPERIMENTAL SETUP

The Oberbeck pendulum is a crossbar, mounted on a double pulley. The axis of the pulley is mounted horizontally and is fixed in the bearings. Crossbar rotates under the force of the tension of the thread, which is wounded on the pulley. The moment of tension force is changed either by usage of cargoes with different mass m, which are attached to the free end of the thread, or by changing the radius of a pulley on which the thread is wounded. The moment of inertia of the instrument is changed by moving the four small bodies of equal mass m along the guide bars of the crossbar at a distance l from the axis of the instrument.

MEASUREMENTS AND DATA PROCESSING

Task 1. Study the dependence of *ε(M)*

Take two sets of measurements of *h* and *t* with several mass values of the cargo suspending to the thread for two different cases. The first case – the thread is wounded on pulley of the radius R_1; the second case - the thread is wounded on pulley of the radius R_2. Calculate the value of ε and M_z. Plot out the dependence of $\varepsilon(M_Z)$. Determine the value of *I* from the plot. Estimate the setup accuracy and the random error of the measurements *I*. Write down values of *t, h,* R_1, R_2, *m,* Δm, M_z, ε and *I* in the table.

Task 2. Study the dependence of *I* on the mass distribution relative to the axis of rotation

Take two sets of measurements of *h* and *t* for two different positions of the load bodies on the crossbar. The first position is equal to the distance l_1; the second - is equal to the distance l_2. Make sure that the setup is balanced (load bodies are arranged symmetrically), while measuring distances l_1, l_2,. Calculate the values of the moment of inertia of the setup I_1 and I_2 from the experimental data points, using the equations (2) and (3). Compare the difference of these values with the theoretical value:

$$I_2 - I_1 = 4m_0(l_2^2 - l_1^2) \qquad (4)$$

Write down values of *h, t, l,* Δm*, m, R* and *I* in the table.

QUESTIONS AND EXERCISES

1. Derive the equations (1) - (4).
2. Why should the load bodies on the crossbar in the experiment be positioned symmetrically relative to its axis of rotation?
3 What is the dominant type of error (random or systematic), which determines the accuracy level of the measurement of *I* in the experiment?
4. What is the necessary accuracy level for the “standard” value *g* for the calculation of *I* ?
5. Derive the equation of the moment of inertia of the crossbar with the load bodies.
6. Calculate the speed of cargo and the angular velocity of the crossbar at the time *t* after the beginning of the movement. Get the answer on the basis of the laws of dynamics and the law of conservation of the energy.