

## Topic 5. Quasi-stationary circuits

### Practical #2.9

#### Study of the resonance of voltages and currents at the industrial frequency

## 1 Introduction

While preparing to this work, one must first study the *Introduction* to the work 2.8.

Connect in series an inductor and a capacitor into the closed circuit to form an oscillatory system (oscillatory circuit). Frequency of the natural oscillations in the circuit  $\omega_0 = 2 \cdot \pi \cdot f_0 = 1/\sqrt{L \cdot C}$  (Thompson's formula), where  $f_0$  – is the natural oscillation frequency in Hz.

The impedance  $Z$  of this circuit at frequency  $\omega$  is equal to

$$Z = \sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2}, \quad (1)$$

where  $R = R_l$  – is the active resistance of the coil,  $\omega$  – is the angular frequency of the AC current flowing in the circuit.

If this circuit is connected in series to the *emf* of the AC current with frequency  $\omega$ , then forced oscillations of this frequency arise in the circuit.

The voltage resonance takes place, when the frequency of the EMF coincides with the frequency of natural oscillations of the circuit. In such case, the voltage across the capacitor and the inductance exceed the source voltage. This phenomena happen due to the fact that at  $\omega = \omega_0$  the impedance of the series circuit is minimal, equal to  $R_l$  and is pure active. Therefore, the current in the circuit is maximal and, consequently, the voltage on its elements is also maximal.

For a circuit with a parallel connection of the variable *emf* source, inductor and capacitor (parallel circuit), the impedance is

$$Z = \frac{1}{\sqrt{\omega^2 \cdot C^2 + \frac{(1 - 2 \cdot \omega^2 \cdot L \cdot C)}{(R_l^2 + \omega^2 \cdot L^2)}}}, \quad (2)$$

In this case, at the resonance frequency, the impedance is maximal and equal to  $Z_{RES} = L/(R \cdot C)$ . At the same time, the current in the circuit supply circuit is minimal and currents resonance is observed when the current in the branches of the circuit exceeds the current value before splitting between the branches. In other words, a "proper" current of large magnitude flows inside the circuit.

The experimental value of  $Z_{RES}$  can be determined from the Ohm law  $Z_{RES} = V_{RES}/I_{RES}$ , where  $V_{RES}$  and  $I_{RES}$  are the biasing voltage and current of the circuit under resonance.

## 2 Experimental tasks

- Measure and plot out the voltage and the current dependencies on the capacitance value in the series and the parallel oscillatory circuits;
- Calculate the parameters of the circuit from the results of the previous measurements;

- Plot out the vector diagrams and the graphical determination of the phase shift of voltages and currents in circuits.

### 3 Experimental setup

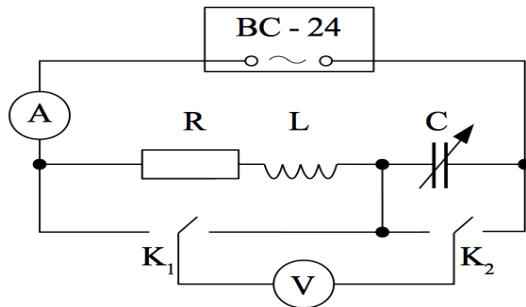


Fig. 9.1

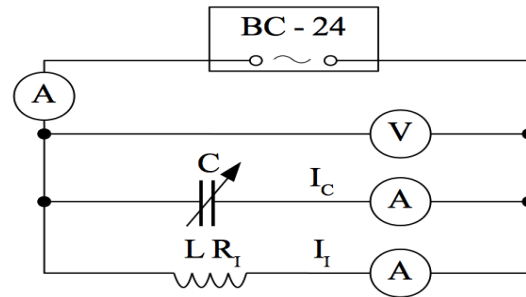


Fig. 9.2

The following elements will be used: a two-section inductor coil, a variable capacitor, three milliammeters, a combined device И4311, as well as two change-over switches. A step-down transformer is used as a source of *emf*. The transformer is the part of the **BC-24** power supply (rectifier).

The setup for studying the voltage resonance is shown in Fig. 9.1. The inductor coil is depicted in it as a series connection of the active resistance of the winding  $R = R_l$  and the ideal inductance  $L$ .

The instrument И4311 is connected to the middle terminals of the keys, which allows using it to measure different voltages. When both keys are set to the left position on the diagram, the voltage on the coil  $V_l$  is measured.

The voltage across the capacitor  $V_C$  is measured when both keys are in the right position. To measure the total voltage  $V$  over the circuit, the left key is set to the left position, while the right key - to the right position.

A setup for studying the current resonance is shown in Fig. 9.2. It allows one to measure the current in each arm of the circuit and the total current in the circuit, as well as the voltage applied to the circuit. In this diagram, the coil is shown as one element with inductance  $L$  and resistance  $R_l$ .

During measurement, the capacitance of the variable capacitor can be changed by means of switches and contact pins. In the initial state, the pins are inserted in the lower row of sockets on the front panel of the variable capacitor. They can be rearranged to the top row sockets. Capacity (in microfarads) is equal to the sum of the digits near the switches and those sockets of the upper row into which the pins are inserted.

Increase in the capacitance of the circuit leads to a decrease in its natural oscillation frequency. At capacitance that corresponds to the oscillation frequency of 50 Hz (the frequency of the urban AC network), one can observe resonant phenomena in the circuit.

### 4 Preparation of protocols

Write down the number and title of the work in the notebook.

Write down the title: "**Calculation formulas**".

Write down the formulas to find:

- the angular frequency:  $\omega_0$  ;
- the impedance of the circuit at resonance:  $Z_{RES} = V_{RES} / I_{RES}$  ;

- the inductance:  $L=1/((\omega_0)^2 \cdot C_{RES})$  ;
- the phase shift between current and voltage on the coil:  $\arctan((\omega_0 \cdot L)/Z_{RES})$  ;
- the impedance of the coil:  $Z_C=V_I/I_I$  ;
- the reactance of the coil:  $X_I=\omega_0 \cdot L$  ;
- the active resistance of the coil:  $R_I=\sqrt{Z_I^2-X_I^2}$  ;
- the theoretical value of the impedance of the parallel circuit:  $Z_{RES}=L/(R \cdot C)$  ;

## 5 Measurements and data processing

After obtaining an admission to work, proceed to perform the experimental tasks.

### 5.1 Task 1. Study of the voltage resonance

Draw the table 1 to record the results of measurements of the current and voltages  $V_I$ ,  $V_C$  and  $V_{tot}$ , which correspond to 10 different capacitance values.

Table 1

# meas	$C, \mu\text{F}$	$I, \text{mA}$	$V_I, \text{V}$	$V_C, \text{V}$	$V_{tot}, \text{V}$
1					
...					
10					

To complete task 1, assemble the setup in accordance with the Fig. 9.1. In this case, the coil should be connected to the circuit with the maximum number of windings, that is, through the end clamps of the panel with the inscriptions 1200 and 2400. Note that the active resistance of the coil is depicted as the resistor  $R = R_I$  in the figure, but it is not an independent element in the setup.

Set all tuning knobs on the variable capacitor to zero, then set 2  $\mu\text{F}$  capacitance limit with the pins.

Set the keys to a position that allows you to measure the total voltage.

Switch the instrument И4311 into the AC voltage measurement mode by pressing the "U ~" button and set the measurement limit of 20V by pressing the corresponding button.

Set the voltage control knob on the source panel to the far left position.

**Attention!** Turning on and tuning the setup is the responsibility of an engineer / teacher. Take the first measurement in engineer / teacher presence.

Write down the capacitance value of the circuit, the current value in the circuit provided by the current source and the voltage  $V$  applied to the circuit.

The further measurements are carried out by the student independently.

Move the left-handed switch to the right position, measure and write the  $V_C$  value in the table. Move both keys to the left position and measure the value of  $V_I$ .

Turn the right-handed switch to the right position, set the variable capacitor to 4 $\mu\text{F}$ . Repeat the measurements and write down the results in the table.

Perform similar measurements, recording the results into the table 1, each time increasing the capacity by 1  $\mu\text{F}$  up to 16  $\mu\text{F}$  inclusively.

Turn off the power supply and the И4311.

## 5.2 Task 2. Study of the resonance of currents

To complete task 2, assemble the setup in accordance with Fig. 9.2. At the same time, an ammeter with the smallest measurement limit must be connected to the power supply.

Prepare the table 2 to record the results of the measurements of  $V_{tot}$ ,  $I_{tot}$ ,  $I_L$  and  $I_C$ , which corresponds to 10 different capacitance values.

Table 2

# meas	$C, \mu\text{F}$	$I_{tot}, \text{mA}$	$V_{tot}, \text{V}$	$I_L, \text{A}$	$I_C, \text{A}$
1					
...					
10					

After checking the scheme by an engineer or a teacher, take a set of measurements of the current before splitting and in each arm of the circuit for 10 different values of the capacitance from 2  $\mu\text{F}$  to 16  $\mu\text{F}$ , maintaining a constant voltage value applied to the circuit. Write down the results into the table 2.

After finishing with all the measurements, turn off the rectifier and the instrument II4311. Put the workplace in order.

## 5.3 Task 3. Processing of the results of the measurements

For a series circuit, plot out the dependences of  $V_{tot}$ ,  $V_L$ ,  $V_C$  and  $I$  on  $C$  on the same sheet. The graph of the total voltage will be a horizontal line, the others will have the form of curves with a maximum.

Calculate the value of the circuit impedance  $Z_{RES}$  at the resonance, the inductance of the coil and the voltage phase shifts on the coil at resonance and at capacitance values of 4 and 10  $\mu\text{F}$ .

Write down the results into the table 3.

Table 3.

Impedance ( $Z$ ), $\Omega$	
Inductance ( $L$ ), H	
Phase shift at resonance (calc), deg	
Phase shift at $C = 4\mu\text{F}$ (calc), deg	
Phase shift at $C = 10\mu\text{F}$ (calc), deg	
Phase shift at at resonance (diagram), deg	
Phase shift at $C = 4\mu\text{F}$ (diagram), deg	
Phase shift at $C = 10\mu\text{F}$ (diagram), deg	

Plot out the vector diagram of voltages at resonance. Read the *Appendix* below before plotting the vector diagrams.

Construct similarly vector diagrams for  $C = 4\mu\text{F}$  and  $C = 10\mu\text{F}$ . Determine the angles of the phase shift of voltages and currents with the protractor and compare them with the calculated data.

For a parallel circuit, plot out the dependences of  $V_{tot}$ ,  $I_I$ ,  $I_C$  and  $I$  on  $C$  on the same sheet. The graph of the total current in the coil will be a horizontal line, the graph of the current for the arm of the circuit with the capacitor will be the rising curve, while the graph of the total current in the circuit will have the form of the curve with the minimum at resonance. This corresponds to the statement that, the current "inside" the resonant circuit is greater than the total current in the entire circuit at resonance.

Calculate the impedance value  $Z_{RES}$  of the circuit at the resonance, the inductance of the coil and the phase shift of the voltage across the coil at resonance and at capacitance values of 4 and 10  $\mu\text{F}$ . Write down the results into the table 4, which is similar to the table 3.

Plot out the vector diagram of voltages at resonance. To do this, first read the *Appendix* to this description.

Plot out the vector diagrams similar to the previous one for  $C = 4\mu\text{F}$  and  $C = 10\mu\text{F}$ . Determine the angles of the phase shift of voltages and currents with the protractor and compare them with the calculated data.

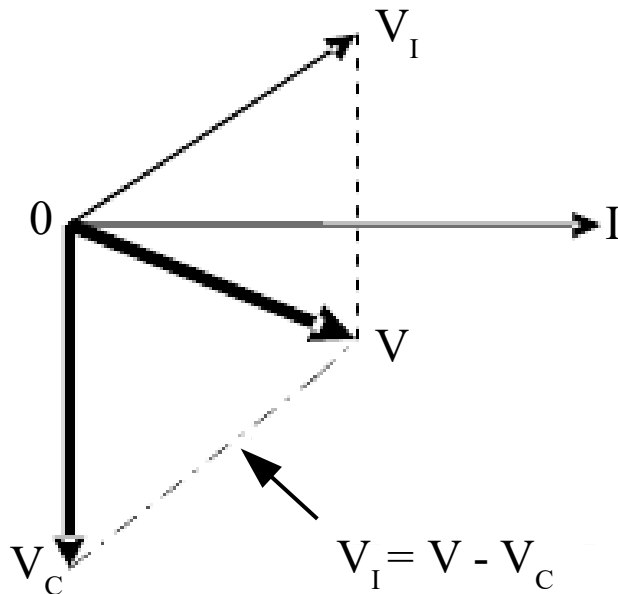


Fig. 9.3

one can separate the vector  $V_I$  along the orthogonal directions  $V_{I_{pure}}$  and  $V_{R_{pure}}$  (see Fig. 9.5), determine the corresponding voltages and calculate the inductive and active resistances of the coil, and also measure (or calculate) the phase shift angles.

The vector diagram for the parallel circuit is constructed in a similar way. But the initial step in this case is the direction of the voltage vector on the circuit, since its value is the same for both branches of the circuit (see Fig. 9.4).

### Appendix

Vector diagrams allow you to graphically determine the parameters of the elements of AC circuits and the phase shift between voltage and current in such circuits, for example, when it is impossible to measure separately the voltage on the inductance and the active resistance of the circuit section, as in the case of a real inductor.

For a series circuit, the vector diagram is constructed from the results of measurements of the voltage on the capacitor  $V_C$ , the coil  $V_I$  and at the ends of the considered circuit segment  $V$ . Since the voltage  $V$  is the vector sum of two other voltages, so the first step is to construct a triangle along three sides with the known direction of the vector  $V_C$  vertically downwards. Then one need to complete it to a parallelogram (Fig. 9.3). After this,

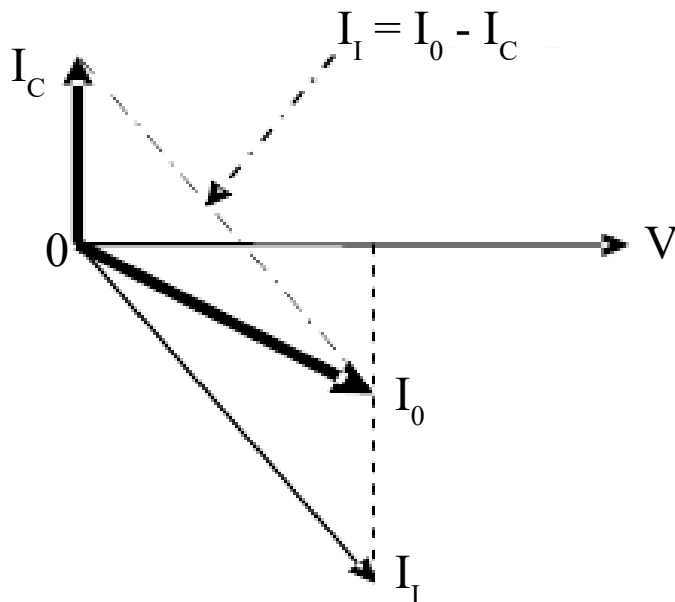


Fig. 9.4

Then the current vector in the capacitive arm  $I_C$  will be directed upwards at an angle of  $90^\circ$ , while the current vector in the inductive branch  $I_I$  will be directed downward at some angle. In other words, the direction of the phase shift of the voltage with respect to the current is maintained regardless of the type of circuit.

Make the necessary plotting steps. Measure the rotation angle of the vector  $I_I$  relative to the horizontal axis with the protractor and compare it with the theoretical value of the phase shift between the voltage on the circuit and the current in the coil.

It is recommended to observe the following sequence of actions (see Fig. 9.5) to plot out the vector diagram for the resonance of voltages:

1. Draw a horizontal line on the sheet of paper that corresponds to the direction of the original current vector whose initial phase is assumed to be zero.
2. On the graphs plotted out from the results of **Task 1**, draw a vertical line for the selected value of capacitance.
3. Measure the voltage  $V_C$  along this line with a divider and set it ( $V_{C\ pure}$  - on the Fig. 9.5) on the vector diagram from the origin vertically downwards.
4. For the same capacitance value, measure the value  $V_{tot}$  with a divider and draw a circle with this radius from the origin.
5. For the same capacitance value, measure the value of  $V_I$  with the divider and make the notch at the intersection with the circle  $V_{tot}$  from the end of the vector  $V_C$ .
6. Connect the intersection point of the circles to the end of the vector  $V_{C\ pure}$ . This is the difference between the vectors  $V_{tot}$  and  $V_C$ , which is equal to  $V_I$ .
7. Transfer the vector  $V_I$  to the origin, that is, complete the resulting triangle to the parallelogram.
8. Measure with the protractor the angle of rotation of the vector  $V_{tot}$  with respect to the horizontal axis and compare it with the theoretical value of the phase shift between the voltage and the current in the circuit.



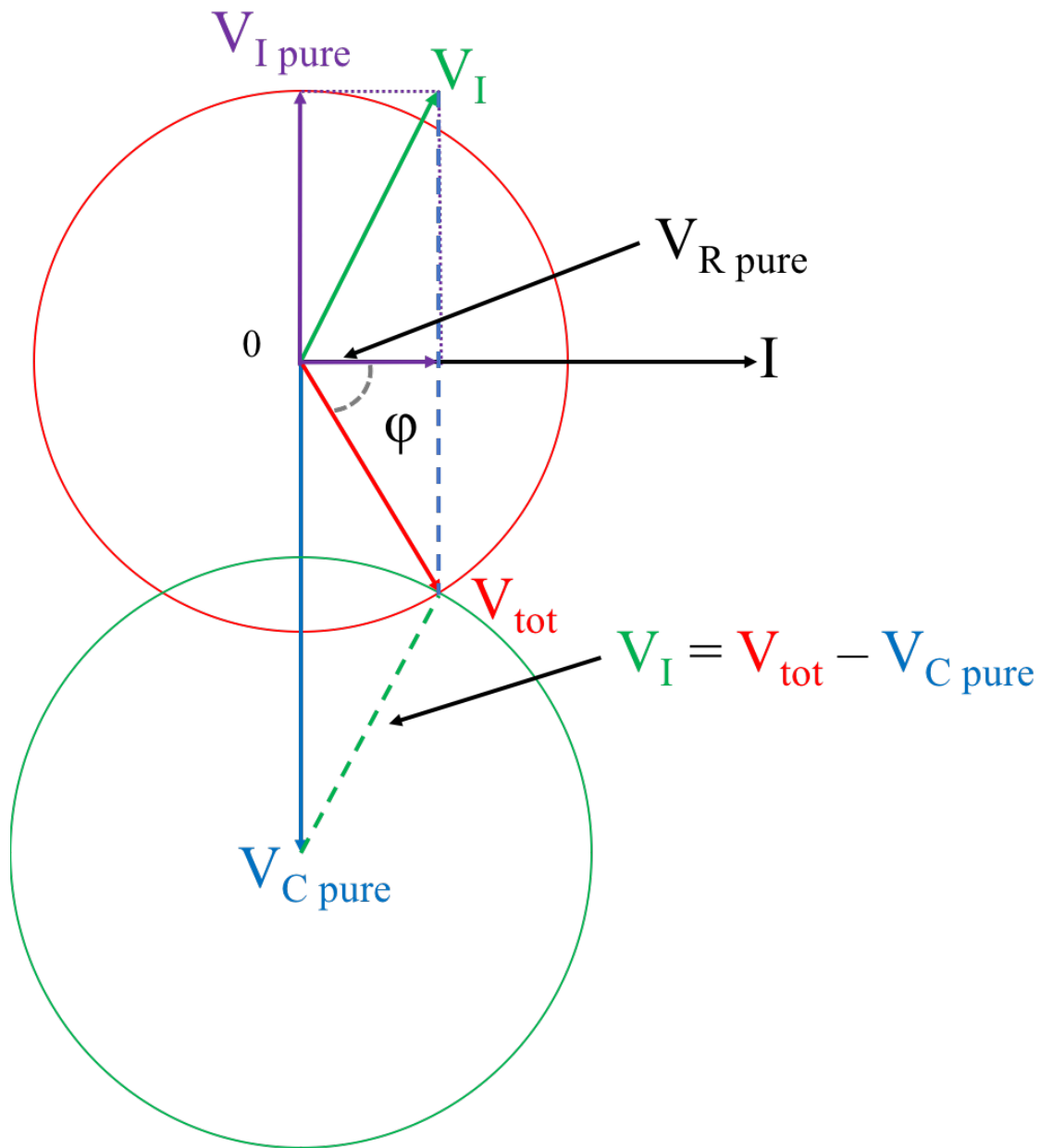


Fig. 9.5

It is recommended to observe the following sequence of actions (see Fig. 9.6) to plot out the vector diagram for the resonance of currents:

1. Draw a horizontal line on the sheet of paper that corresponds to the direction of the original voltage vector whose initial phase is assumed to be zero.
2. On the graphs plotted out from the results of **Task 1**, draw a vertical line for the selected value of capacitance.
3. Measure the voltage  $I_C$  along this line with a divider and set it ( $I_{C \text{ pure}}$  - on the Fig. 9.6) on the vector diagram from the origin vertically upwards.
4. For the same capacitance value, measure the value  $I_I$  with a divider and draw a circle with this radius from the tip of the  $I_{C \text{ pure}}$  vector.



5. For the same capacitance value, measure the value  $I_{tot}$  with a divider and draw a circle with this radius from the origin
6. Connect the intersection point of the circles to the end of the vector  $I_C$ . This is the difference between the vectors  $I_{tot}$  and  $I_{C\ pure}$ , which is equal to  $I_I$ .
7. Transfer the vector  $I_I$  to the origin, that is, complete the resulting triangle to the parallelogram.
8. Measure with the protractor the angle of rotation of the vectors  $I_{tot}$  and  $I_I$  with respect to the horizontal axis and compare it with the theoretical value of the phase shift between the voltage and the current in the circuit and on the inductor.

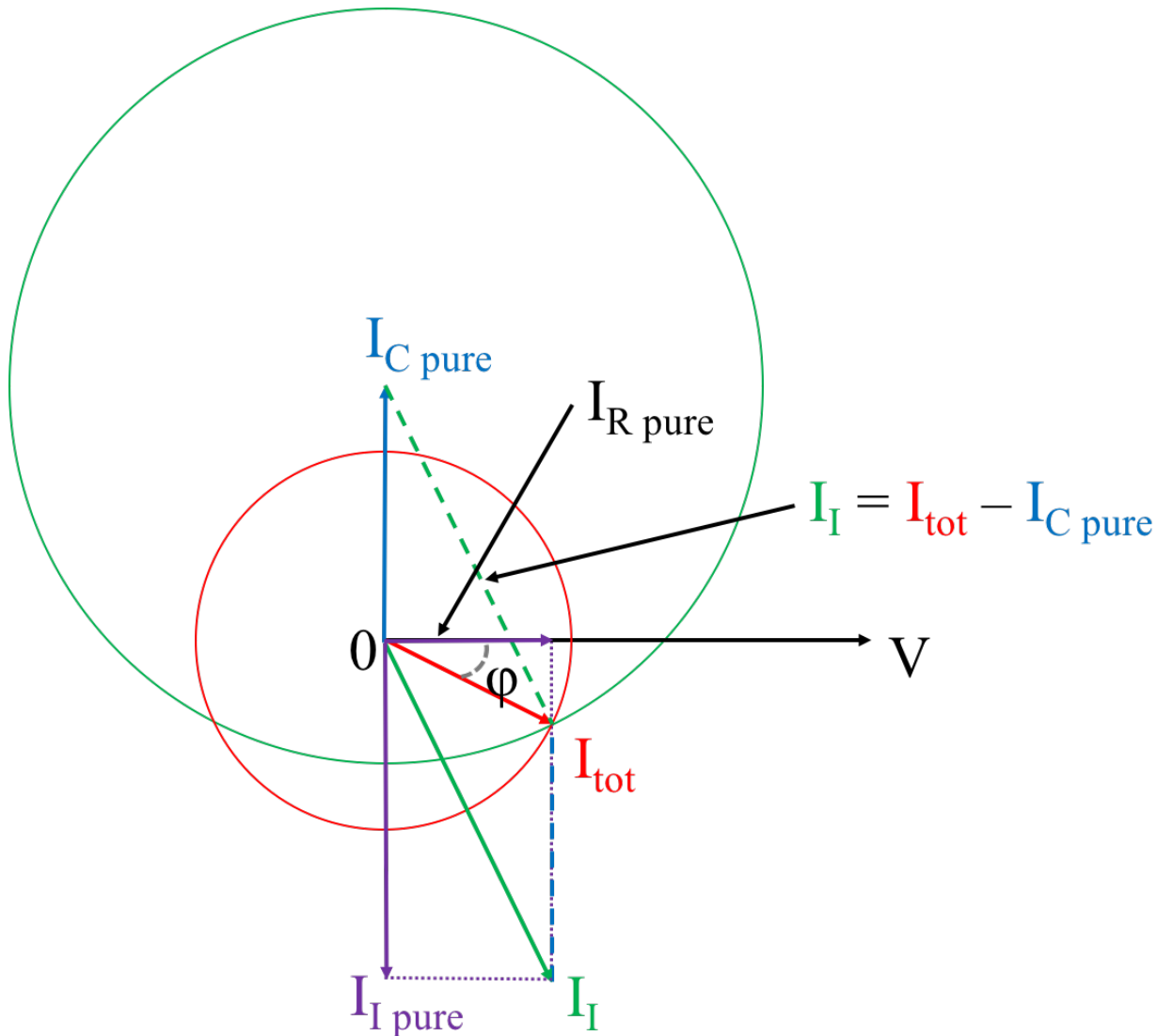


Fig. 9.6

## 6 Questions

1. Formulate the condition of quasi-stationarity of the chain.
2. What is the effective value of the current, voltage?
3. Which voltage value (current value) is measured by a voltmeter (ammeter)?
4. Derive the formula 3 from the laboratory work #2.8.

5. What phenomena is responsible for the appearance of a phase shift in a circuit, which contains capacitance, inductance.
6. Which elements of the circuit have active resistance, and which - reactive resistance?
7. What is the phase shift between the current and voltage in the circuit with the active resistance, with the ideal inductor; with a real inductor; with an ideal capacitor?
8. Is the impedance of the coil the same for DC and AC current, respectively?
9. What is the ratio between inductive and capacitive resistances at resonance?
10. What phenomena is called the voltage resonance, under what conditions is it observed?
11. Explain why the voltages on the reactive elements of the series circuit at resonance exceed the total voltage on the circuit.
12. What is the voltage at the active resistance of the series circuit at resonance, if the supply voltage of the circuit is, for example, 10V?
13. What is Q-factor of the oscillatory circuit? On what values does it depend?
14. How will the width of the resonance curve change with the decrease of a) the active resistance, b) the inductance, c) the capacitance of the circuit?
15. What is the resonance resistance of a series circuit?
16. What phenomena is called the current resonance, under what conditions is it observed?
17. Draw an equivalent circuit of a capacitor with a "bad" dielectric.