Topic 5. Quasi-stationary circuits

Practical #2.8
Ohm's law for the alternating current (AC) circuits

1 Introduction

The processes in electric circuits powered from AC *emf* sources will be studied in this practical work. The *emf* sources called the AC sources, if their output voltage is governed by the harmonic law:

$$\varepsilon=\varepsilon_m\cdot\cos(\omega t+\varphi_0), \tag{1}$$

where ε - is the instantaneous value of the *emf,* ε_m - is ε maximum value (amplitude), ω - is the angular frequency equal to $2\pi/T$ (T - is the oscillation period), and φ_0 - is the initial phase.

If the period of variation of the *emf* and the ones of the current and voltage in the circuit is much longer than t - the propagation time of the electromagnetic field along this circuit, then it can be considered with a high degree of accuracy that the values of these quantities at any time in all sections of the successive circuit are the same. Therefore, chains, which meet the condition T >> t, are called quasi-stationary.

Multiplying both parts of such inequality by the speed of light, one can rewrite the condition of quasi-stationarity in the following way:

$$c\cdot T\gg c\cdot t \quad \text{or} \quad \lambda\gg l, \tag{2}$$

For the AC circuit operating at industrial frequency of 50 Hz, this condition is met if the circuit size is much less than $6\cdot10^6$ m. Thus, even within a megacity like Moscow, such curcuits can be considered quasi-stationary with a high degree of accuracy.

The fulfillment of the condition of quasi-stationarity means that while calculating such circuits one can use the laws for the direct current, for example, Ohm's law, etc., of course, taking into account the peculiarities of propagation of the AC current.

These features are largely related to the properties of electrical circuit elements (along with the resistance *R*), such as capacitance *C* (the ability to accumulate electrical charges) and inductance *L* (the ability to create a magnetic field around itself).

Elements whose behavior is described by one of the listed parameters are called ideal. Such elements are resistors *R* and capacitors *C*. Inductors are not ideal because they are characterized by two parameters: the inductance *L* and the resistance R_I (the resistance of the wire with which they are wound).

However, the presence of a capacitance and even an ideal inductance (R_I = 0) is equivalent to the appearance in the AC circuits of additional resistance. In contrast to the resistance *R*, which is called active, since it irretrievably transforms the electric current energy into thermal energy, the capacitance reactance and the inductance reactance are called reactive. The term "reactive" is used to point out the fact that in such elements the energy of the electric current is not only spent on the charge of the capacitor or the creation of a magnetic field, respectively, but also completely returns to the electric circuit when the capacitor is discharged or the magnetic field disappears. The reactance resistances of the capacitance and inductance are defined as X_C and X_I, respectively, and are equal to:

$$X_C=1/(\omega\cdot C) \quad \text{and} \quad X_I=\omega\cdot L \quad . \tag{3}$$

The complex-valued generalization of resistance (the impedance) of the portion of the AC circuit includes all active and reactive resistances.

In most practical cases, it is traditional to specify the so-called effective value of the AC (and voltage) instead of the amplitude values. It coincides in magnitude with the strength of the direct current, which in this circuit creates a thermal effect equal to the effect created by the AC current. It can be shown that $I_{eff}=I_m/\sqrt{2}$ or $I_{rms}=I_{peak}/\sqrt{2}$ and $V_{eff}=V_m/\sqrt{2}$ or $V_{rms}=V_{peak}/\sqrt{2}$.

Then Ohm's law for AC circuits can be written as $Z=U_{eff}/I_{eff}=U_m/I_m$. The voltage of 220V of a conventional AC power supply network is the effective voltage. The corresponding magnitude value is about 311V. The vast majority of electrical measuring instruments are graduated in effective values. Therefore, the magnitude of the impedance can be determined experimentally by the ammeter-voltmeter method. Further in the text, the indices "*eff*" and "*m*" will be skipped in all subsequent formulas and vector diagrams.

The presence of capacitance and inductance in AC circuits also leads to the appearance of a phase shift between the current and voltage: the voltage on the inductor outpaces the current by $\pi/2$, while it is behind the current by $\pi/2$ on the capacitor.

The vector diagram method is often used to solve practical problems for calculating AC circuits. This method is identical to the one, which is used in mechanics to find the result of adding of oscillations of the same frequency ω occurring in the same direction (see **Appendix** to laboratory work 1.9).

The following equation $V=\sqrt{V_R^2+(V_I-V_C)^2}$ comes from the vector diagram for the segment of the circuit which includes a capacity, inductance and resistance connected in series. Then, using Ohm's law, one can obtain

$$V=I\cdot\sqrt{R^2+(X_I-X_C)^2}=I\cdot\sqrt{R^2+(\omega\cdot L-1/(\omega\cdot C))^2}=I\cdot Z \quad . \tag{4}$$

As follows from (4), the impedance of such segment is equal to

$$Z=\sqrt{R^2+(\omega\cdot L-\frac{1}{\omega\cdot C})^2} \quad . \tag{5}$$

Thus, measuring the voltage and current in the circuit one can determine the impedance Z, which later can be used to calculate the unknown values of capacitance or inductance.

It also follows from the vector diagram that the phase shift φ between the current in the circuit and the voltage in the general case is equal to

$$\varphi=\arctan[(X_I-X_C)/R] \quad . \tag{6}$$

The phase shift between current and voltage can be determined experimentally using an oscilloscope. To do this, one need to measure the period and the displacement of the corresponding sine waves relative to each other along the horizontal axis in units of the scale of the oscilloscope, and then expresses the phase shift in angular units, taking into account that the period corresponds to a phase shift of 360° (or 2π radians).

2 Experimental tasks

- Determine the capacitance of capacitors;
- Determine the inductance of the sections of the inductor coil;
- Determine the impedance of an unbranched segment of the circuit containing capacitance and inductance;
- Determine the phase shift between voltage and current in the circuit.

3 Experimental setup

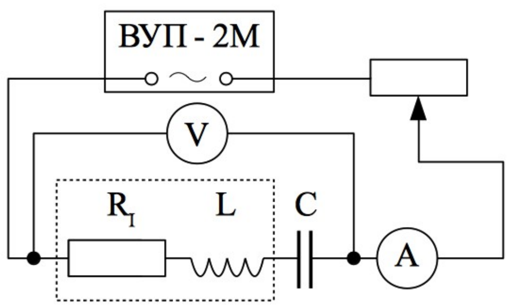

Fig. 8.1

The setup for the measurements is assembled by the student himself in several variants, one of which is shown in Fig. 8.1.

Depending on the purpose of the measurements, various research objects are connected in parallel with the voltmeter. These objects are: two capacitors with different capacitance, multi-section inductor (coil) and unbranched circuit made up of one coil section (in Figure 8.1 it is represented as two ideal elements (R_I and L) and is surrounded by a dashed line) and one of the capacitors. A current source of the type **ВУП - 2М** is used, which allows obtaining an adjustable AC voltage of the industrial frequency of 50 Hz at the output. The current in the circuit is adjusted by the rheostat. A multimeter V is used to measure the voltage, the multimeter A (Ц 4311) is used to measure the current, the oscilloscope C1-65A is used to observe the phase shift.

4 Preparation of protocols

Write down the number and title of the practical work.

Write down the title: ***"Calculation formulas"***.

Write down the title: ***"Task 1. Measurement of the capacitance"***.

Write down the formulas for calculating: the angular frequency ω, the impedance Z (according to the Ohm law), the capacitance C (assuming in the expression for Z that $R = L = 0$), and also formulas for calculating of the total capacitance of two capacitors connected in parallel and in series.

Write down the title: ***"Task 2. Measurement of the inductance"***.

Write out the formulas for calculating the reactance of the coil X_I and its impedance Z_I, knowing ω and its L and R_I and assuming $C = 0$ in the expression for Z.

Write down the title: ***"Task 3. Measurement of the impedance of the circuit"***.

Write out the formula for calculating the theoretical value of the impedance Z of the circuit from the results of measurements of R, X_I and X_C, where R is the sum of the active resistance of the coil R_I and the resistance of the "active" part of the rheostat R_R (see Figure 8.1).

Write down the title: ***"Task 4. Determination of the phase shift between the voltage and the current in the circuit"***.

Write out the formula for calculating the phase shift between the current and the voltage at the ends of the considered circuit, as

$$\varphi = \arctan[(X_I - X_C)/(R_I + R_R)] \quad . \tag{7}$$

5 Measurements and data processing

After obtaining a theoretical admission to work, proceed to its implementation.

5.1 Task 1. Measurement of the capacitance

Write the subtitle: ***"Task 1 Measurement of the capacitance"***
Prepare the table 1.

Table 1

	I, mA	V, V	$Z_C = X_C$, Ω	C, μF
C_1				
C_2				
C(parallel)				
C(series)				

To perform **Task 1**, connect the rheostat, ammeter and one of the capacitors (C_1) in series. Connect this circuit to the output terminals "~" of the source. Set on the panel of the ammeter an AC measuring mode with a measuring limit of 7.5 mA. Set the rheostat wiper to the position of highest resistance.

After checking the scheme by an engineer or teacher, turn on the source and the ammeter. Set the rheostat wiper so that the needle of the ammeter is closer to the end of the scale. (Reminding: it reduces the measurement error). Turn on the multimeter. Set the multimeter to the AC operation mode with the measurement limit of 20V. Measure the current and the voltage on the capacitor C_1. Put down the results into the table 1 and turn off the source.

Attention. Turn off the power supply after each measurement!

Calculate the capacitance of the capacitor C_1.

Connect the second capacitor C_2 instead of C_1 and perform the second set of measurements and calculations.

Connect capacitor C_1 parallel to capacitor C_2 and perform the third set of measurements and calculations.

Connect the capacitors in series and perform the fourth set of measurements and calculations.

Calculate the theoretical values of the total capacitance of capacitors for various connection schemes, using the previously obtained capacitance values of each of them separately. Write them in the notebook and compare the theoretical values with the experimental ones.

5.2 Task 2. Measurement of the inductance

Write the subtitle: ***"Task 1 Measurement of the inductance"***
Prepare the table 2.

Table 2

Section	R_l, Ω	I, A	V, V	Z_l, Ohm	X_l. Ω	L, H
1200						
2400						
3600						

Measure with the help of the multimeter the active resistances of the coil sections (the number of turns in each section is indicated on the contact plates of the coil and in the 1st column of Table 2) and the whole coil and write them down into table 2.

Connect the section of the coil, which marked on its terminal strip as "1200", instead of the capacitors and measure the current and the voltage in the same way as in **Task 1**.

Connect the second coil section instead of the first section and perform similar measurements.

Connect the entire coil and perform similar measurements.

Based on the results of each measurement, calculate the corresponding inductance values.

5.3 Task 3. Measurement of the impedance of the circuit

Write the subtitle: ***"Task 1 Measurement of the impedance of the circuit"***

Prepare the table 3.

Table 3

Current (I), A	
Voltage (V), V	
Experimental impedance value (Z_{exp}), Ω	
Capacitance of the capacitor (C), µF	
Rheostat resistance (R_R), Ω	
Active coil resistance (R_l), Ω	
Active circuit resistance (R), Ω	
Reactive coil resistance (X_l), Ω	
Reactive capacitor resistance (X_C), Ω	
Reactive circuit resistance (X), Ω	
Theoretical impedance value (Z_{theor}), Ω	

Insert the capacitor with a higher capacitance value in series in the circuit, which was used in the **Task 2**.

Measure the current and the voltage at the source terminals. Turn off the source.

Calculate and write down into the table 3 the experimental value of the impedance (obtained by the ammeter-voltmeter method).

Disconnect one of the terminals of the rheostat and measure with the help of the multimeter the resistance of the used part of the rheostat and write its value into the table 3. Write down the impedance value of the circuit elements defined in the previous tasks into the table 3. Use the data from the table 3 to calculate active and reactive resictance and the impedance of the circuit.

5.4 Task 4. Determination of the phase shift between the voltage and the current in the circuit

Write the subtitle: ***Task 4 “Determination of the phase shift between the voltage and the current in the circuit”***

Prepare the table 4.

Table 4

Theoretical phase shift, deg	
Phase shift from the triangle of resistance, deg	

Calculate the phase shift between the current and the total voltage, using the data from the table 3, and write down the result into the table 4.

Plot the triangle of resistance in the notebook with the scale of 1 cm = 100 ohms. Determine the angle between X and Z with a protractor and write it down into the table 4.

6 Questions

1. Formulate the condition of quasi-stationarity of the chain.
2. What is the effective value of the current, voltage?
3. Which voltage value (current value) is measured by a voltmeter (ammeter)?
4. Derive the third formula from the practical work #2.8.
5. What phenomena is responsible for the appearance of a phase shift in a circuit, which contains capacitance, inductance.
6. Which elements of the circuit have active resistance, and which - reactive resistance?
7. What is the phase shift between the current and voltage in the circuit with the active resistance, with the ideal inductor; with a real inductor; with an ideal capacitor?
8. Is the impedance of the coil the same for DC and AC current, respectively?
9. What is the ratio between inductive and capacitive resistances at resonance?
10. What phenomena is called the voltage resonance, under what conditions is it observed?
11. Explain why the voltages on the reactive elements of the series circuit at resonance exceed the total voltage on the circuit.
12. What is the voltage at the active resistance of the series circuit at resonance, if the supply voltage of the circuit is, for example, 10V?
13. What is Q-factor of the oscillatory circuit? On what values does it depend?
14. How will the width of the resonance curve change with the decrease of a) the active resistance, b) the inductance, c) the capacitance of the circuit?
15. What is the resonance resistance of a series circuit?
16. What phenomena is called the current resonance, under what conditions is it observed?
17. Draw an equivalent circuit of a capacitor with a "bad" dielectric.