Topic 4. Magnetic phenomena
Practical #2.7
Study of the magnetic field

1 Introduction

The magnetic field **B** is defined through the force that it exerts on a moving charged particle. Experiment shows that this force is proportional to the charge q of the particle and its velocity **v**:

$$\mathbf{F}=k_1 q\mathbf{v}\times\mathbf{B}. \quad (1)$$

The coefficient k_1 in Eqn (1) depends on the choice of units. It is equal to $1/c$ (where c is the speed of light in vacuum) in Gauss's system and 1 in the International System of Units (SI). In the laboratory we tend to use SI units because they are more practical than Gaussian units.

Recalling the definition of the current density from hydrodynamics, we can introduce the electric-current density

$$\mathbf{J}=\rho\mathbf{v}=en\mathbf{v}, \quad (2)$$

where e is the elementary charge, n is the number density of charged particles, and **v** is the so-called 'drift velocity' of charged particles at a given point of space at a given time (as in hydrodynamics, **v** is not the velocity of a specific particle, but a velocity field). Combining (1) and (2), we can write an expression for the force acting on a current-carrying wire in a magnetic field (*Ampere's force*):

$$\mathbf{F}=k_1 I l\hat{\mathbf{v}}\times\mathbf{B}, \quad (3)$$

where I is the current carried by the wire, l is the length of the segment of the wire in hand.

It was found experimentally that a current-carrying wire not only experiences the effect of a magnetic field, but itself creates a magnetic field. It is this field that Oersted observed when he placed a compass needle near a current-carrying wire.

When a medium is placed in a magnetic field, it responds by becoming *magnetised*. Computing the magnetic field in a medium is in general an intractable problem because of the enormous number of particles involved. The way round this difficulty is to introduce the auxiliary field **H** (called the magnetic field strength, or magnetic field intensity) that is generated by *free currents*, i.e. currents that flow in wires and that are considered given. The **B**-field (called the magnetic induction, or the magnetic flux density) is generated by all the currents flowing through the medium: the free currents and the currents induced by the magnetic field.

In the simplest case of *linear medium* the **B**-field and **H**-field are proportional:

$$\mathbf{B}=k_2\mu\mathbf{H}, \quad (4)$$

where k_2 is another constant that depends on the choice of units, and μ is the magnetic permeability of the matter. In Gauss's system $k_2 = 1$, in the SI it is designated $\mu_0 = 4\pi \times 10\text{-}7$ Wb A^{-1} m^{-1}.

2 Experimental tasks:

- measuring the intensity of the Earth's magnetic field at the observation point (Moscow);

- measuring and calculating the magnetic induction of the field created inside the solenoid;
- study of the theoretical and experimental dependences of the magnetic induction on the position of the measurement point inside the solenoid and on current in its winding.

3 Experimental setup

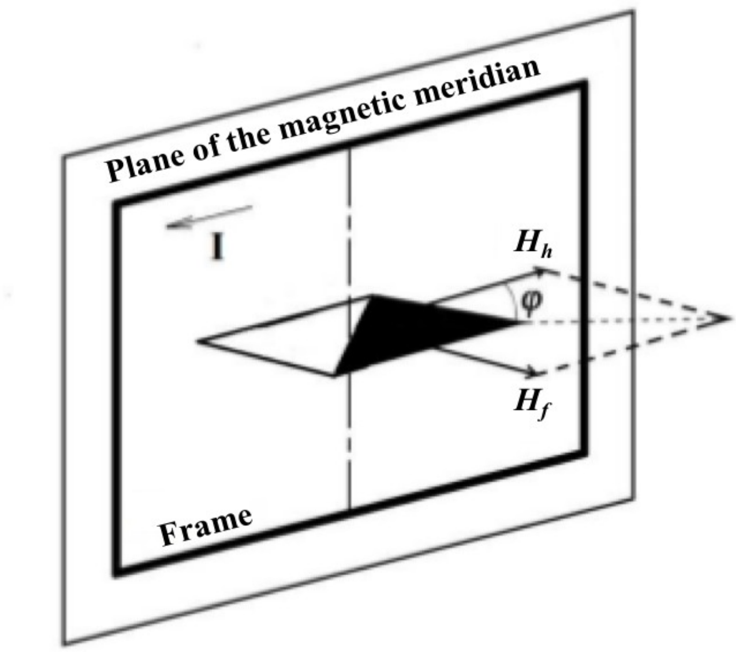

Fig. 7.1

Several different setups will be used in this work.

The horizontal component of the H-field (H_h Fig. 7.1) of the earth's magnetic field is measured by means of a tangent-galvanometer (Fig.1). Tangent-galvanometer is an instrument which consists of a vertical frame with the current and a compass right in its center. The compass is equipped with a dial-scale for counting the rotation angle of the needle, while its magnetic needle is capable to rotate in the horizontal plane.

The frame is located in the plane of the magnetic meridian. The needle will turn to some angle φ, under the influence of the H-field (H_f Fig. 7.1) created by the frame. The lines of force generated by the frame are perpendicular to its plane.

Then $H_h = H_f / \tan(\varphi)$. The following equation can be used to calculate H_f:

$$H_f = (2\sqrt{2} \cdot N \cdot I)/(\pi \cdot L) \quad (5)$$

where N – is the number of windings in the frame, I – is the current in the frame, L – is the length of the side of the frame. The frame is connected to the DC current source (BC - 24M) through the dual switching key **K** (Fig. 7.2). It allows to revert the direction of the current in the frame, to exclude the error, which is related to the uncertainty of the initial position of the frame. The current I in the frame is measured by a milliampere meter (**mA**) and can be adjusted with a rheostat (**R**).

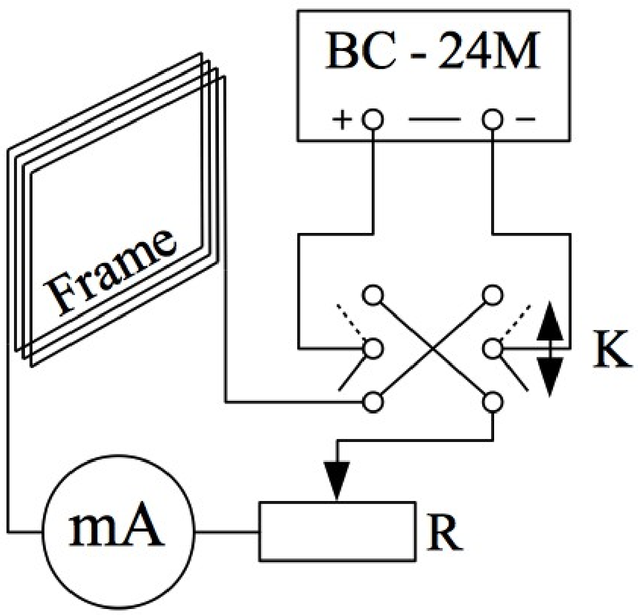

Fig. 7.2

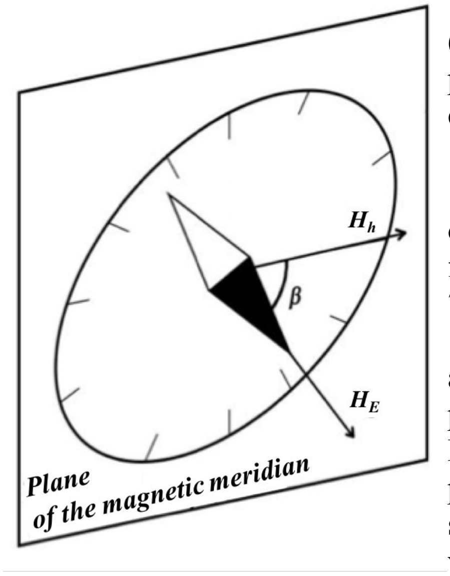

Fig. 7.3

The angle of magnetic inclination β is measured with a declinator (see Fig. 7.3), which is also consists of a magnetic needle on the axis, provided with a circular scale, so the plane of the needle with the scale can be set both in the horizontal and in the vertical plane.

It is obvious that the intensity of the Earth's magnetic field is $H_E = H_h / \cos(\beta)$. For Moscow $H_h = 16\,A/m$. The value of β depends on the geographical latitude of the observation site, changing from zero at the equator to 90 ° at the magnetic pole. For Moscow, β = 70°.

The magnitude of the magnetic induction at various points along the axis of a long coil with a current (solenoid) are measured in the second part of the work. For this purpose, a special device - fluxmeter (FM) (see Fig.7.4) is used to measure the magnetic flux *Φ*. It is connected to the probe coil (PC), which measures the flux *Φ* generated by the section the solenoid S. The value of B is determined from the relation $\Phi = B \cdot S \cdot N$, where *Φ* is the magnetic flux penetrating the section of the probe coil of area *S*; *N* is the number of windings in the probe coil.

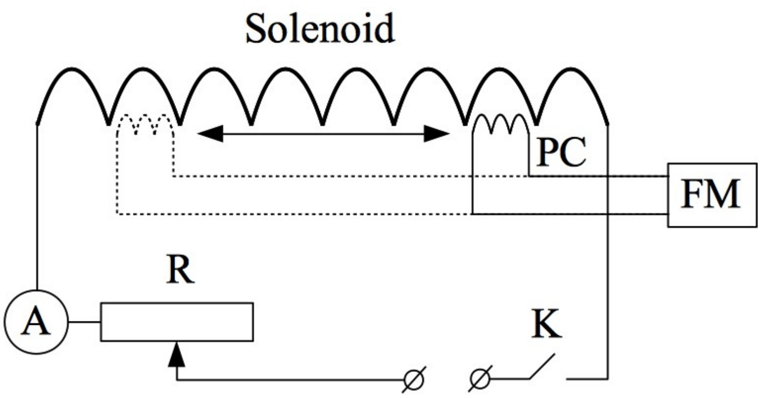

Fig. 7.4

The probe coil is fixed to the movable rod and placed inside the solenoid. The position of the coil is determined by the ruler, along which the arrow is moved. The arrow itself is attached to the rod.

The magnetic flux changes by ΔΦ = Φ (from 0 to Φ and vice versa), when the current is turned on (or turned off) through the solenoid. Due to the law of electromagnetic induction, this leads to the appearance of the *emf*

$$\varepsilon = d\Phi / dt\,, \quad (6)$$

and, as the result, to the current in the measuring circuit $I = \varepsilon / R$ (where *R* is the resistance of the measuring circuit). Substituting $\varepsilon = I\,R = R\,dq/dt$ into (2) and integrating this expression with respect to time, we obtain $q = \Delta\Phi / R = \Phi / R$, in other words, the magnetic flux in the probe coil and the charge flowing in it are proportional to each other.

The fluxmeter measures instantaneous values of the induction current, its integration and recalculation of the charge into the magnetic flux, the values of which are shown on the scale of the device.

4 Preparation of protocols

Put down the number and title of the work into the notebook.

Put down the title: **"Calculation formulas"**.

Put down the subtitle: **"Task 1. Measurement of the Earth's magnetic field strength"**.

Write out the formulas (with a description of the notations) for calculating H_E, H_h, and also H_f in the form of $H_f = K_1 \cdot I$, where $K_1 = (2\sqrt{2} \cdot N) / (\pi \cdot L)$. When performing calculations, you must first calculate K_I based on the passport data of the setup and then use it in subsequent calculations, which will accelerate then significantly.

Write down the formula for calculating the relative instrumental error of the H_h measurements: $\varepsilon_H=((\Delta I/I)^2+(\Delta\varphi/\cos^2(\varphi))^2)^{0.5}$, where ΔI and $\Delta\varphi$ are the instrument errors in the measurement of the current intensity and the deflection angle of the magnetic needle of the compass, respectively.

Put down the subtitle: "**Task 2. Measurement and calculation of the induction of the magnetic field created inside the solenoid"**.

Write down the calculation formula for the experimental values of B:

$$B_{\exp}=\Phi/(S\cdot N), \qquad (7)$$

Write down the calculation formula for the magnetic induction:

$$B_{theor}=\mu\cdot\mu_0\frac{n\cdot I}{2}\cdot\left(\frac{l-x}{\sqrt{R^2+(l-x)^2}}+\frac{x}{\sqrt{R^2+l^2}}\right), \qquad (8)$$

where n - is the number of windings per unit length of the solenoid; I - is the current in the solenoid coil, l - is the length of the solenoid; x - is the distance from the edge of the solenoid to the point under study; R - is the radius of the solenoid.

Rewrite (4) in the form $B_{theor}=K_2\cdot\left(\frac{l-x}{\sqrt{R^2+(l-x)^2}}+\frac{x}{\sqrt{R^2+l^2}}\right)$ and separately the expression for

$K_{theor}=\mu\cdot\mu_0\frac{n\cdot I}{2}$. As with task 1, this form allows you to speed up your calculations.

Write (4) in the form $B_{theor}=K_3\cdot I$ and separately the expression for

$$K_3=\mu\cdot\mu_0\frac{n}{2}\cdot\left(\frac{l-x}{\sqrt{R^2+(l-x)^2}}+\frac{x}{\sqrt{R^2+l^2}}\right) \text{ for } x = 1/2.$$

Use this equations for subsequent calculations.

5 Measurements and data processing

After obtaining a theoretical admission to work, proceed to its implementation

5.1 Task 1. Measurement of the Earth's magnetic field strength.

Prepare the table 1 for recording the results of the H_h measurements.

Table 1

I, A	φ_{left}	φ_{right}	φ_{aver}	$\tan(\varphi_{aver})$	H_h, A/m	$\cos(\varphi)$	ε

Write out the passport data of the setup (N and L), which are required to perform the ***Task 1***.

Assemble the measuring circuit in accordance with the Fig. 7.2.

After checking by a teacher or an engineer, set the plane of the frame in the plane of the magnetic meridian, in other words, along the magnetic needle.

Turn on the power source and turn key K to the left position. Using a rheostat, set the current so that the magnetic needle deflects to an angle of 35°. Write down the obtained values of the current and the deflection angle of the needle into the table 1. Turn the key to the right position and count the angle of deflection of the needle to the other side. Carry out the same measurements for the angles of 45° and 50°.

Calculate the value of H_h for each measurement and the average value that is used in subsequent H_E calculations.

Write out the values of ΔI and $\Delta\varphi$ and calculate the relative error ε_H for each measurement.

Write down the value of the angle φ for which the relative error ε_H is minimal.

Prepare the table 2 for recording the declinator readings.

Table 2

# measurement	β(north)	β(south)	β_{aver}
Average value of β_{aver}			

To reduce the influence of external magnetic fields, place the declinator on the top shelf of the laboratory table away from other devices, especially from voltage sources. Set the plane of the declinator horizontally. Turn the declinator in the horizontal plane, set the zero scale division opposite to the north end of the magnetic needle.

Turn the plane of the declinator to the vertical position. Count the angle β according to the positions of the north and south ends of the needle. Write down the average of these values.

Carry out three similar measurements in different places on the table. As a final result, take the average of the obtained values.

Calculate the H_E module.

5.2 Task 2. Measurement and calculation of the induction of the magnetic field created inside the solenoid

Assemble the power supply circuit of the solenoid (Figure 7.4) by connecting the power supply, solenoid, rheostat, key and ammeter in series.

Turn on the power of the solenoid and set the current equal to 1A. Open the key in the solenoid supply circuit. Prepare the table 3.

Table 3

x, cm	Φ, μWb			B_{exp}, T	B_{theor}, T
	On	Off	Aver.		
1					
...					
l					

Set the probe coil to the far left position (x = 1).

Invite the engineer to check the circuit, obtain a practical admission and set up the fluxmeter (switch on the device and set the zero point). In this case, the "limit" switch should be in the 2500 μWb position.

Take the first series of measurements in the presence of the engineer.

Switch off "ARR" the button. Press the "MEASUREMENT" button.

Switch the solenoid power key to the “short” position. Record the result on the scale of the instrument and put it down into table 3 into the "On" column.

Switch the solenoid power key to the “open” position. Record the result on the scale of the instrument and put it down into table 3 into the "Off" column. Push the "ZERO" button.

Perform similar measurements along the entire length of the solenoid. In this case, it is recommended to change the position of the probe coil in the interval from 1 to 10 cm every 1cm, in the interval from 10 to 40 cm - every 10 cm, in the interval from 40 to 50 cm - every 1 cm. Press the "ARR" button, after finishing all the measurements.

Place the probe coil to the middle position and prepare table 4 for recording the results. This table is similar to the table 3 with the difference that in the first column you need to write instead of the x coordinate the magnitude of the current I in the solenoid, which must be changed from 1A to 0A every 0.1A. Measure Φ for all current values.

Attention! After each measurement, it is necessary to press the “**ARR**” button before changing the current in the solenoid.

Write down the passport data of the solenoid (n, l, R).

Calculate the average value of the magnetic flux, and then the theoretical and experimental values of B and write them down into the table 3.

According to the data in the table 3, plot out the dependences of the theoretical and experimental values of B on x on one sheet and on I on the other.

Appendix

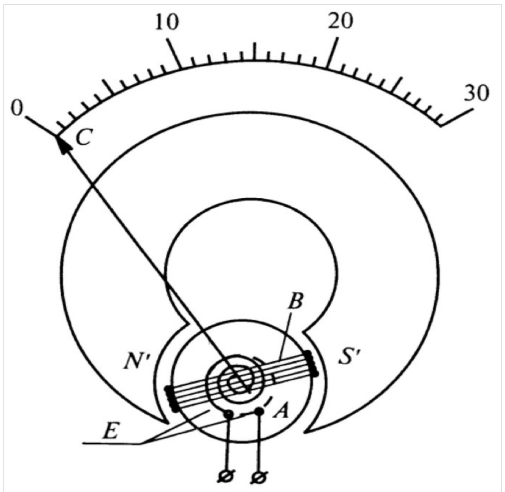

Fig. 7.5

The forces of magnetic interaction have found wide application in practice, in particular, in the construction of electromeasuring instruments.

In the moving coil system, the movable part of the measuring mechanism (Fig. 7.5) consists of the B – a plane rectangular coil (wire wound on a light rectangular frame), which can rotate in the gap between the fixed steel cylinder A and the permanent magnet poles ($N'S'$). The frame is fixed on two semiaxes, and the pointer (needle) C is fixed to it.

A permanent magnetic field with induction B is created in the gap between the magnet tips and the steel cylinder. The ends of the winding of the coil are connected to the coil springs E, through which the measured current is fed.

The interaction of the magnetic field and the frame with the current in it (Ampère force) leads to the appearance of a moment of force, turning the frame and the arrow on it. The magnitude of this moment is proportional to the current in the frame.

The rotation of the frame causes the twisting of the springs E, in which the counteracting moment of the elastic forces M_p arises. This moment is proportional to the twisting angle (rotation of the frame) α.

When these moments are equal, the pointer stops. In this case, $\alpha = C_I I$, where C_I is the instrument constant, which depends on the instrument design and determines the value of division of the instrument. Therefore, the magnitude of the current can be determined from the angle of rotation α of the measuring mechanism.

The magnetic field in the gap where the current frame rotates is usually large enough, so even a weak current causes a significant torque, while the external magnetic fields practically do not introduce errors into the measurement result.

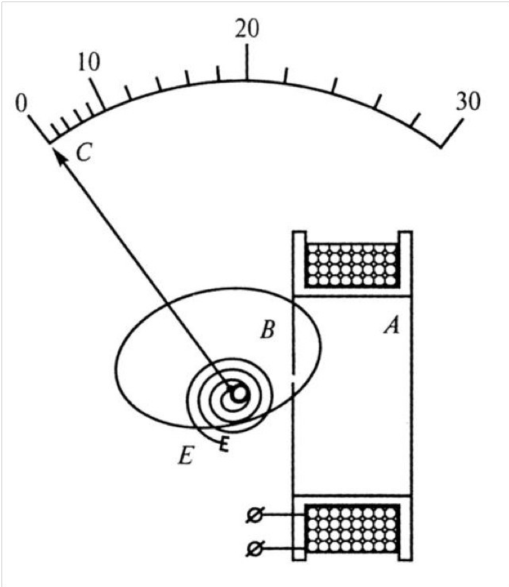

Fig. 7.6

The advantages of the instruments with the moving coil system include: the uniformity of the scale; high sensitivity; low power consumption from the measured circuit; insensitivity to external magnetic fields. To drawbacks are the following: the ability to measure only direct current; sensitivity of the measuring mechanism to electrical overloads and external mechanical influences.

The measuring mechanism of the moving iron system (Fig. 7.6) consists of a stationary coil *A* and a movable ferromagnetic core *B*, eccentrically fastened on one axis with the pointer of the instrument *C*. One of the ends of the spiral spring *E* is attached to the same axis. It creates the counteracting moment forces while the spring is twisted.

Under the influence of the magnetic field created by the measured current *I* flowing through the coil, the core rotates drawing into the coil and tends to settle, so that the energy of the system is minimal. Since the energy of the magnetic field of the coil is proportional to the square of the current flowing through it, we can assume that the resulting torque is dependent on the current strength quadratically.

If the moment, which generates by an interaction of the magnetic field and the current, is equal to the moment of the elastic forces of the spring *E*, then the needle stops. In this case $\alpha = C_2 I^2$, where C_2 is the device constant, which depends on the instrument system and determines its minor division value of the scale. As the result, the magnitude of the current can be determined from the rotation angle of the measuring mechanism α. Since the rotation angle of the needle of the device α is proportional to the square of the current, the scale of the instruments of this system is uneven, unlike the moving coil ones.

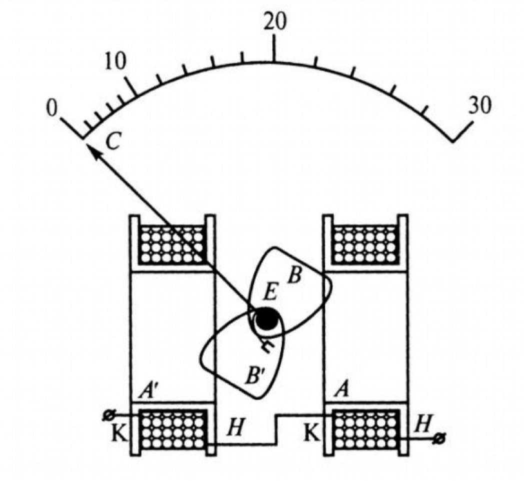

Fig. 7.7

Since the rotation angle of the needle of the device is independent from the direction of the current (since α is proportional to the square of the current strength), such devices are suitable for measuring both direct and alternating current.

The measuring mechanism of such devices proved to be very sensitive to external magnetic fields penetrating into the coil, since the field of the coil can be quite small. Therefore, to protect from external magnetic fields, either an iron screen or an instruments with the astatic mechanism are used.

In the astatic instruments (Fig. 7.7), there are two identical coils located on both sides of the axis with two cores.

The coils are wound so that the field of one is directed opposite the field of the other. The external field, amplifying the action of one coil, weakens the effect of the other and as a result has practically no effect on the readings of the instrument.

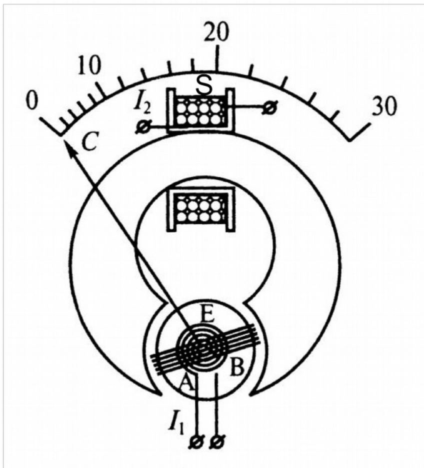

Fig. 7.8

The construction of instruments with the electrodynamic system is very close to the instruments with the moving coil system with an exception that the magnetic field, in which the frame with current *B* is turned, is created by the fixed coil S (Fig. 7.8).

In this case, the torque acting on the moving coil depends both on the magnitude of the current in the fixed coil I_1 and on the current in the moving coil I_2: $M = K \cdot I_1 \cdot I_2 \cdot \cos(\varphi)$, where φ is the phase shift between the currents I_1 and I_2. Just like in other instruments, the spiral spring E provides the appearance of an opposite moment of forces, which leads to proportionality of the rotation angle α of the coil B, which is proportional to the product of $I_1 \cdot I_2$. If the coils are connected in series and fed with the measured current, then the angle of rotation of the needle will be proportional to the square of the current.

Thus, the instruments with the considered systems are inherently ammeters.

But the instruments of the electrodynamic system can also be used to measure the useful power released in the circuit (load). In this case, the winding of the fixed coil (usually its resistance R_L is much greater than the load resistance R) is connected in parallel to the load, as a voltmeter. A moving coil (whose resistance R_M is usually less than R) connected in series with the load, as an ammeter. Then the rotation angle of the needle will be proportional to the power consumed by the load, since $\alpha \sim I_1 \cdot I_2 = (U / R_L) \cdot I_2 \sim U \cdot I_2 = P$.

6 Questions

1. How can one find out the direction of the lines of force of a magnetic field created by a conductor with a current?
2. How can one determine the direction of the magnetic field lines produced by the coil (frame or solenoid), knowing the direction of the current in it?
3. How can one determine the direction of the magnetic field lines knowing the position of the magnetic needle?
4. Draw the picture of the lines of force of the Earth's magnetic field and show on it how does the value of the angle of magnetic inclination change with the change of the coordinates of the observation point?
5. Draw a schematic view of the Earth from the pole, indicate the positions of the geographic and geomagnetic poles on it and show how does the angle of the magnetic declination changes with the change of the coordinates of the observation point (latitude - when moving along the radius and longitude - when moving around the circle)?
6. Draw a picture of the lines of force of the magnetic field produced by the solenoid.
7. What is the physical meaning of B and H? In what units are they measured?
8. What is the magnetic permeability μ?
9. How do substances are classified in respect to the μ value?
10. Which ferromagnets are called magnetically soft?
11. What ferromagnets can be used to fabricate permanent magnets?
12. What is the Lorentz force?
13. What is Ampère's force?
14. Formulate the law of electromagnetic induction.
15. How would the needle of the electrical measuring instrument be positioned in the absence of coil springs?
16. Why the instruments with the moving coil system can not be used with alternating current circuits?
17. How can one compensate an influence of external magnetic fields on the instruments with the moving iron system?
18. How can one determine to which system the instrument belongs to, based on the symbols on the scale of the instrument?
19. Why does the scale of the instruments with the electrodynamic system irregular for measurements of the current intensity, while it is uniform for the measurements of power?
20. Why does the instrument to the electromagnetic system have the irregular scale?