# Practical 2.3

# Modeling the electrostatic field

## INTRODUCTION

The field or space around a charged particle (so called electrical charge) where its force can be experienced by any other charged particle is called the *electric field* of the former charge. *Electric field* is used to describe forces of electrical interaction. The electric field, $\vec{E}$, at a given point is defined as the (vector) force, that would be exerted on a stationary test particle of unit charge by electromagnetic forces (i.e. the Lorentz force). A particle of charge $q$ would be subject to a force $\vec{F} = q \cdot \vec{E}$. From this definition, the SI units of the electric field $\vec{E}$ are N/C, or V/m, or using the base SI units kg·m/s$^3$/A.

If an electric field is created by stationary charges, then it is called electrostatic, that is, not changing in time. An electric field is characterized by its magnitude (as defined above) or by its *electric potential* ($\varphi$) which is the energy characteristic of the field. It is equal to the potential energy of the test charge placed at a given point of the field. It is the amount of work needed to move a unit positive charge from a reference point to a specific point inside the field without producing any acceleration. Typically, the reference point is Earth or a point at the infinity, although any point beyond the influence of the electric field of the charge can be used.

Difference of the potentials between two points of the field is called the *electric voltage*. The magnitude of the voltage does not depend on the absolute value of the potential. Therefore, for a zero potential level, the potential can be taken at any convenient point of the field. Electric field (i.e., its magnitude) and potential are related by the relationship: $\vec{E} = -grad\ \varphi$. In other words, the electric field vector is aimed in an opposite direction of the most rapid potential increase. The magnitude equals to the potential difference upon unit length. Movement of charges due to the influence of the electric field is similar to movement of the bodies under the influence of the gravity.

The potential distribution in space is represented by *equipotential surfaces* or surfaces of equal potential, and in case of a two-dimensional (plane) field - in the form of *equipotential lines*. They are similar to lines of equal height on geographical maps. The electric field is represented in the form of lines of force that are perpendicular to the equipotential lines. Tangents to that lines show direction of forces acting on positive charges in the electric field, and the density of the lines is proportional to the magnitude of the field.

These forces also provide influence on the charges in a conductive medium between the electrodes to which a certain voltage is applied. The identity of the solutions of the corresponding mathematical equations under identical boundary conditions provides a mathematical basis for modeling the electrostatic field by the field of a stationary current.

Since it is very difficult to determine the magnitude of the charge and the parameters of the electrostatic field by direct measurement methods, a voltage drop (potential difference) may be measured between any two points in the system through which a flow of the stationary current is provided. It is convenient to study electrostatic fields by modeling in an electrolytic bath filled with conductive liquid (electrolyte). In the electrolyte, metal electrodes are placed, the shape of which corresponds to the shape of the charged electrodes of the two-dimensional field under investigation. Electrodes are supplied with an electric voltage and a series of measurements of the potential difference *U* between one of the electrodes and various points in the electrolyte is carried out. Connecting points with the same values of *U*, we can construct a picture of equipotential lines. The resulting pattern can be scaled both geometrically and by the magnitude of the applied voltage and used to calculate electric fields with a similar geometry of the electrodes.

***Experimental tasks:***

- Plot out equipotential lines corresponding the potential distribution in the electrolyte bath equipped with 3 different electrodes;
- Calculate characteristics of the electric field with use of the measurement results.

## EXPERIMENTAL SETUP

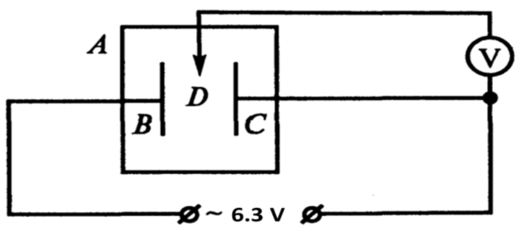

Figure 3.1. Experimental setup

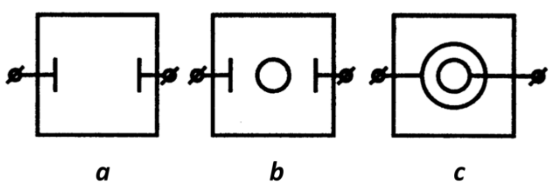

Figure 3.2. Experimental configurations

The experimental setup is depicted in Fig. 3.1. It is a flat bath *A* made of a dielectric. Tap water (distilled water is a dielectric) is used as a conductive medium (electrolyte). Water is poured into the bath in a thin layer so that the model can be considered two-dimensional, that is, viewed as a cross-sectional model of a volumetric electrode system. Then the plane of the bath can be considered as the plane of symmetry of real electrodes.

The bath should be installed strictly horizontally so that the conductivity of the water layer is the same anywhere. On the side walls of the bath there are two vertical stands for fixing and connecting electrodes *B* and *C*, as well as a socket for connecting the probe - a sharp metal rod equipped with a dielectric handle. In this work, several electrode systems are studied (Figure 3.2): two parallel straight electrodes, which are a model of a flat capacitor or deflecting plates of electron-beam tubes; a metal ring placed between the electrodes makes it possible to study the behavior of conductors put into an electric field and their effect on the characteristics of the field; two ring electrodes are a model of a cylindrical or (in the first approximation) spherical capacitor. Each of the electrodes is equipped with a vertical stand for its attachment and connection. There are also two metal rods with clamps at the ends for connecting the pillars of the electrodes to the bath posts. Electrodes lean against the bottom of the bath, towering above the surface of the water. They are connected to an AC source to avoid field distortion due to the electrolysis of water and the formation of gas bubbles near the electrodes. To measure the voltage between the electrode *C* and any point *D* in the electrolyte (Figure 3.1), a probe with a voltmeter connected to it is inserted into this point. In this work, an electronic voltmeter with a high (~ 1 MΩ) resistance is used to avoid the influence of the device resistance on the potential distribution in the bath.

***Formulas:***

$\vec{E}=(\varphi_2-\varphi_1)/d$, with $\varphi_2$ and $\varphi_1$ are measured potentials, *d* – the distance.

Surface charge density:

Flat electrodes: $\sigma = E\ \varepsilon\ \varepsilon_0$, with *E* – the magnitude of the electric field, *ε* – dielectric permittivity, $\varepsilon_0$– electric constant.

Cylindrical electrodes: $\sigma_1=U\cdot\varepsilon\cdot\ \varepsilon_0/(R_1\cdot ln(R_2/R_1))$, $\sigma_2 = U\cdot\varepsilon\cdot\varepsilon_0/(R_2\cdot ln(R_2/R_1))$, with $R_2$ and $R_1$ – the radii of the cylinders, *U* – the potential difference between the electrodes.

***Preparing:***

Pour tap water into the bath. The thickness of the water layer should be the same everywhere and be 3 to 5 mm so that the resistance of the water layer is sufficiently large and the fluctuations in the thickness of the water layer in the bath do not affect the measurement results. Insert the tip of the wire from the probe into the socket on the edge of the bath. Turn on the toggle switch "power" on the panel of the voltmeter. The device should be warmed up for 5 minutes before the measurement starts.

## MEASUREMENT AND DATA PROCESSING

**Task 1.** ***Equipotential lines for the flat capacitor (Fig. 3.2a)***

Install on the bottom of the bath symmetrically about its edges two straight electrodes parallel to each other at a distance of 6 - 8 cm. Connect the racks electrodes and a rack of a bath by horizontal rods and fix clips in the necessary position. Make sure that the electrodes rest on the bottom of the bath with the entire bottom surface, and that they do not have air bubbles. If necessary, remove them with a touch of your fingers. Put on the graph paper the position of the electrodes and the axis of symmetry of the bath. Touch the probe electrode *C* and set the voltmeter's pointer to zero. Touch the electrode of electrode *B*, record the readings of the device on the graph paper next to the image of this electrode. If the pointer does not deviate, check and, if necessary, tighten all electrodes' connections. Lower the tip of the probe into the water vertically in the center of the bath. Without removing the probe from the water, but also without pressing it tightly to the bottom of the bath (to avoid scratches and distortion of the field), move it along the horizontal axis line towards electrode *C* until the instrument reading is 1 V. Label this point on the graph paper. Move the probe 1 cm up and, shifting it to the right-left, find the position where the voltage will also be 1 V. Label it on the graph paper. Again move the probe 1 cm up and find the same voltage point. Continue similar measurements to a point with a coordinate exceeding the edge of the electrodes by 2 to 3 cm. Do the same measurements in the lower half of the bath. Plot a line through all the points of the same voltage on the graph paper. Write down the corresponding value of the potential near it. Find the point on the horizontal axis where the voltage will be 2 V. Construct similarly the equipotential line for this case. Repeat the measurements and build lines in the space between the electrodes for voltages 3 V, 4 V, etc. Make sure that the lines in the central region between the electrodes are parallel, and near the edges of the electrodes they begin to curve around them.

***Task 2. Equipotential lines for the isolated ring. (Fig. 3.2b)***

Install a small ring electrode in the center of the bath. It should not be attached and connected electrically to the power supply. Apply on the 2nd sheet of the graph paper the bath axis and note the position of the electrodes. Measure the potential in the center of the ring, on the ring itself and build a central and 2-3 side equipotential lines in the same way as was done in Task 1.

***Task 3. Equipotential lines for the cylindrical capacitor. (Fig. 3.2c)***

Remove flat electrodes. Install and connect a large ring and small ring electrodes symmetrically to the center of the bath. Apply on the 3rd sheet of the graph paper the bath axis and note the position of the electrodes. Construct equipotential lines between the electrodes. Present the results to the teacher and get his signature on each sheet. Turn off and disassemble the setup. Collect water from the bath and demonstrate the setup to the engineer.

***Task 4. Calculation of the electric field for the model systems studied.***

On each of the resulting drawings of equipotential lines draw several lines representing the electric field and indicate the direction of it. Indicate in the figure with flat electrodes 3-4 points, including near the electrodes and calculate the values of the electric field magnitude.

***Task 5. Calculation of the surface charge density.***

Calculate the surface charge density module, which must be accumulated on flat and cylindrical electrodes, so that they create the same field strength between electrodes in distilled water with a permittivity $\varepsilon = 81$. For cylindrical electrodes, measure and write the electrode radii in the notebook.