PRACTICAL 10

MODELLING THE TUNNEL EFFECT

Objective: Investigation of the basic laws of the tunnel effect using the microwave model.

Experimental equipment: microwave oscillation generator, horn antennas, polystyrene prisms, microwave detector, measuring device (microammeter); tunnel diode type ZI-201G; milliammeter (50 mA); voltmeter (2B); power supply (1.3V)

INTRODUCTION

The tunnel effect is a quantum-mechanical phenomenon that has no direct analogue in classical mechanics. This effect is that a microparticle with energy E can pass through a potential barrier whose height is $U > E$.

The tunnel effect is observed in the following physical phenomena:

- electron tunneling at the contact of two metals, leading to the appearance of a contact potential difference;
- cold emission of electrons from metals in a strong electric field;
- alpha decay of radioactive nuclei (due to the sub-barrier pass of alpha particles through the barrier caused by the Coulomb field).

In addition, the tunnel effect plays an important role in the Josephson effect, and is responsible for the “quantum evaporation” of black holes.

Quantum-mechanical calculation shows that when the height of energy barrier U is greater than the energy E of the particle, there is a finite probability of the particle passing through such a barrier. To characterize this probability, we introduce the concept of transparency of the barrier D, equal to the ratio of the number of particles that have passed the barrier to the total number of particles falling on the barrier.

For a free electron and a rectangular barrier of width d, the transparency is:

$$D = 16\,\frac{E}{U}\left[1-\frac{E}{U}\right]\cdot exp\left[-\frac{2d}{h}\sqrt{2m_e\left(U-E\right)}\right]. \qquad (1)$$

Let’s consider a phenomenon that is similar to the quantum-mechanical tunnel effect. If an electromagnetic wave falls on the interface between two media (Fig. 10.1.) with the refractive index n1 n_1 и n_2 ($n_1 > n_2$) at an angle φ such that

$$sin\ \varphi \geq \frac{n_2}{n_1} = n_{2,1}\ , \qquad (2)$$

then it does not pass into the second medium and the phenomenon of total internal reflection is observed. Theoretically, this phenomenon was considered by Russian physicist Eichenwald. He concluded that the intensity of the wave transmitted into medium 2 varies with the distance x in this medium according to the law:

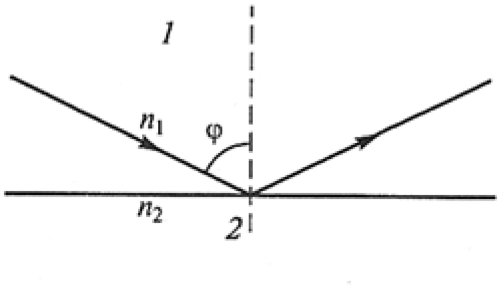

Fig. 10.1.

$$I = I_0 \exp\left(-\frac{2\pi x}{\lambda_1}\sqrt{\sin^2 \varphi - n_{2,1}^{\,2}} \right), \tag{3}$$

where I_0 – intensity at the interface 1 and 2; φ – angle of incidence;

$n_{2,1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ – relative refractive index of two media; λ_1– wavelength in medium 1, from which the wave falls on the interface.

According to Eichenwald's theory on full internal reflection, the field enters the second medium, since it is not interrupted at the interface. Therefore, *the reflection of an electromagnetic wave occurs in a some layer*. The presence of such a layer was demonstrated in optics by Wood, Mandel'shtam and Zeleni, who provided experiments on the observation of a wave transmitted through a thin layer of matter under conditions of total internal reflection.

It is almost impossible to measure the amplitude (or intensity) of a wave beyond the interface with full internal reflection at the optical wave range, but at the centimeter waves (at ultrahigh frequencies) these measurements are relatively easy. For the first time such experiments at a wavelength of 15 cm were made by the Indian physicist Bose. A similar technique is used in the present work.

DESCRIPTION OF EXPERIMENTAL SETUP

The scheme of the measurement setup is shown in fig. 10.2, a general view of the experimental setup - in Fig. 10.3. Its main parts are a microwave oscillator Г (λ= 3cm), a power supply unit БП (DC power supply B5-49), a gate B that prevents the reflected waves come back into the generator, an attenuator A to adjust the power of the outgoing microwave radiation, horn antennas P, with which the radiation is output from the waveguide and fall to the detector D. Registration is carried out using a microammeter “μA”. Under the experimental conditions, it can be considered that the current in the circuit is proportional to the power of microwave radiation incident on the detector. Two triangular prisms P1 and P2 made of polystyrene are placed on the table between the transmitting and receiving horns. Prisms can be moved apart using a special screw. The thickness of the air gap x between the prisms is measured on a scale printed on the table.

The material of the prism is the medium with a coefficient ε_1. The air layer is the second medium with a coefficient ε_2. The radiation falls on the face of the first prism at an angle φ = 450, which is larger than the critical one, therefore full internal reflection should occur on this face. If the second prism is moved a considerable distance away from the first prism, then the incident wave will be completely reflected from the face of the first prism. If the prisms are shifted so tightly that in practice they are a uniform solid cube, then the intensity of the transmitted wave should be approximately equal to the intensity of the incident wave.

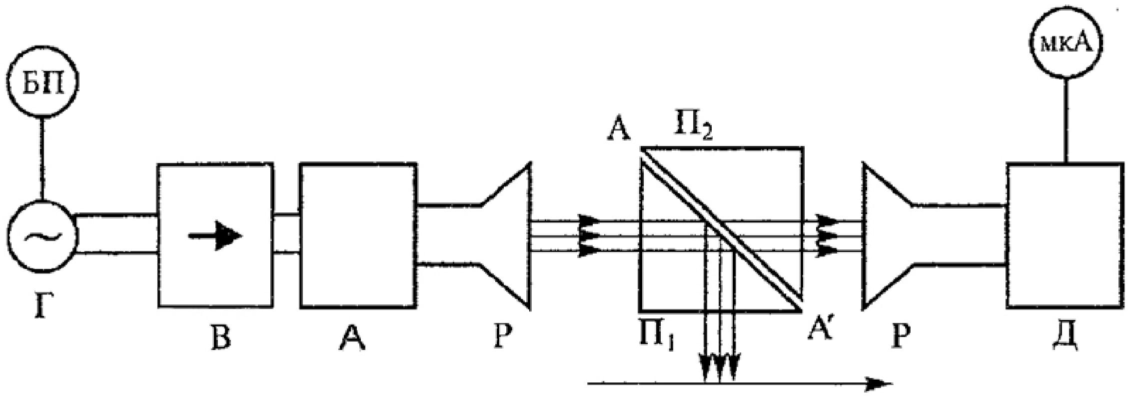

Fig. 10.2.

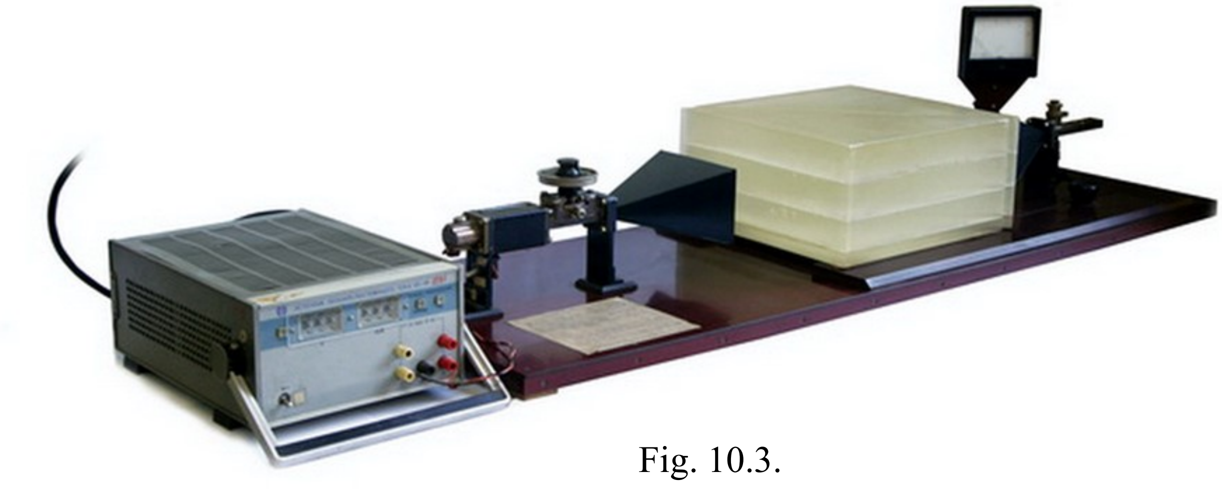

Fig. 10.3.

When we move apart the prisms, while the thickness of the air layer between them is small, a significant part of the wave still passes through the interface and can be registered by the detector.

MEASUREMENT AND PROCESSING OF RESULTS

Task 1. **Measurement of the intensity of the transmitted wave depending on the thickness of the air gap**

1. Remove the protective cover from the prisms and put a protective metal shield in front of the experimenter’s prisms.

2. Turn on the DC power supply B5-49 (toggle switch 1) first, and then the power supply of the generator (toggle switch 2).

3. When prisms are tightly pressed to each other, turn the attenuator knob (Figure 10.4) to achieve the current value 85- 90μA.

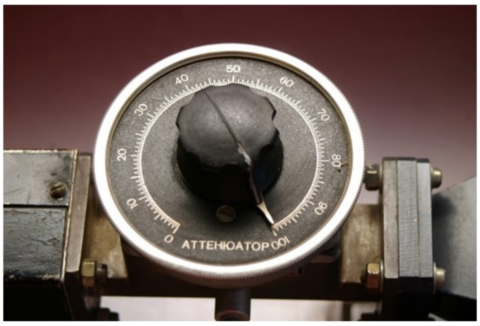

Fig. 10.4.

4. Changing the thickness of the air layer x between the prisms by a screw, measure the dependence of the intensity of the transmitted wave on the layer thickness with a step of 0.1 cm; 0.2 cm and 1 cm at intervals of $0 < x < 1$ cm, $1 < x < 3$ cm and $3 < x < 10$ cm, respectively.

5. At the end of the measurement, turn off both toggle switches and close the prisms with a protective cover.

6. Draw a dependency graphs $I = f(x)$ and $ln \frac{I_0}{I} = f(x)$ in a convenient scale for analyzing the results. Explain the resulting dependency.

Task 2. **Calculation of the dielectric constant of the prism material**

1. Using the Eichenwald formula and taking the wavelength in polystyrene equal $\lambda_1 = \frac{\lambda_2}{\sqrt{\varepsilon_1 / \varepsilon_2}}$, get an expression to calculate the dielectric constant of polystyrene. The wavelength in air is $\lambda_2 = 3$ cm, the dielectric constant of air $\varepsilon_2 = 1$.

2. Using the graphs $I = f(x)$ and $ln \frac{I_0}{I} = f(x)$, as well as the resulting expression for ε_1, calculate the dielectric constant of polystyrene.

3. Compare the experimental value ε_1 with the table (ε_{pol}=2,56).

QUESTIONS AND EXERCISES

1. What is the tunnel effect?

2. Name the phenomena for the explanation of which the tunnel effect should be used.

3. Justify the analogy between the tunnel effect and the phenomenon of total internal reflection of electromagnetic waves at the interface.

4. Write the Schrödinger equation : common and for stationary states.

5. What restrictions are imposed on the wave function, which is a solution to the Schrödinger equation?

6. Solve the problem of a particle incident on a rectangular one-dimensional potential barrier. What is the difference between the behavior of classical and quantum particles incident on a barrier of height U with the energy $E<U$; $E>U$?

7. Find an approximate solution for the transparency of a rectangular one-dimensional potential barrier with a height of U. Estimate the barrier transparency for the electron and proton, if $E-U = 1$ eV, the barrier width is a) $l = 0.1$ nm; b) 1 nm.