- Part II.

Math methods for data analysis:

interpolation and approximation methods

II.2 Interpolation using Newton’s formula

***Interpolation* – a general term (concept) of constructing new data points within the range of a discrete set of known data points**

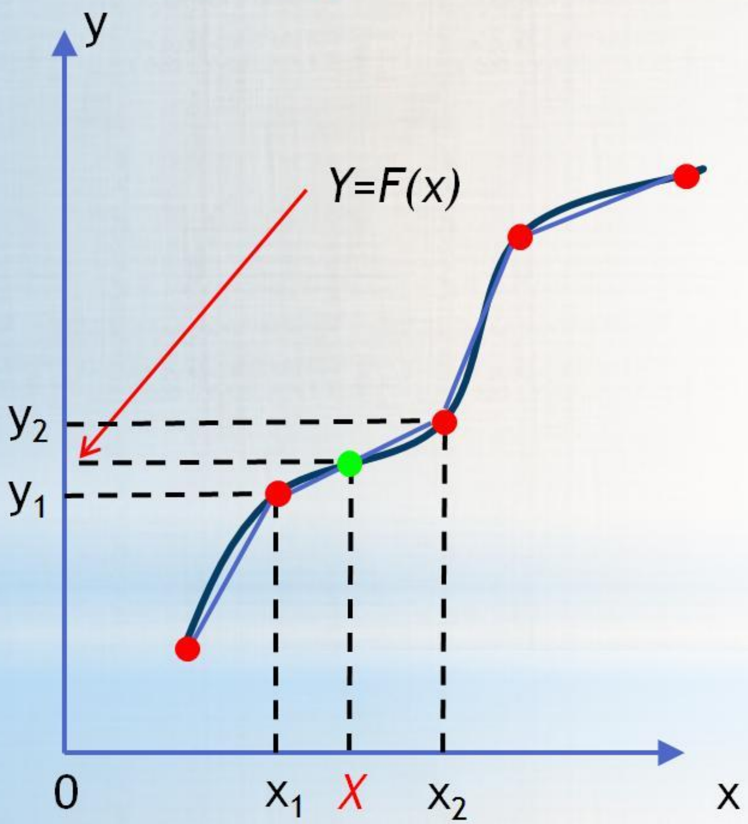
- ***the given data points* are considered to be exact nodes of the function to be constructed**

***(unlike Interpolation), Approximation* – a concept of finding the best possible fit to the given data points, which would allow to predict values outside of the measured range of a discrete set of known data points**

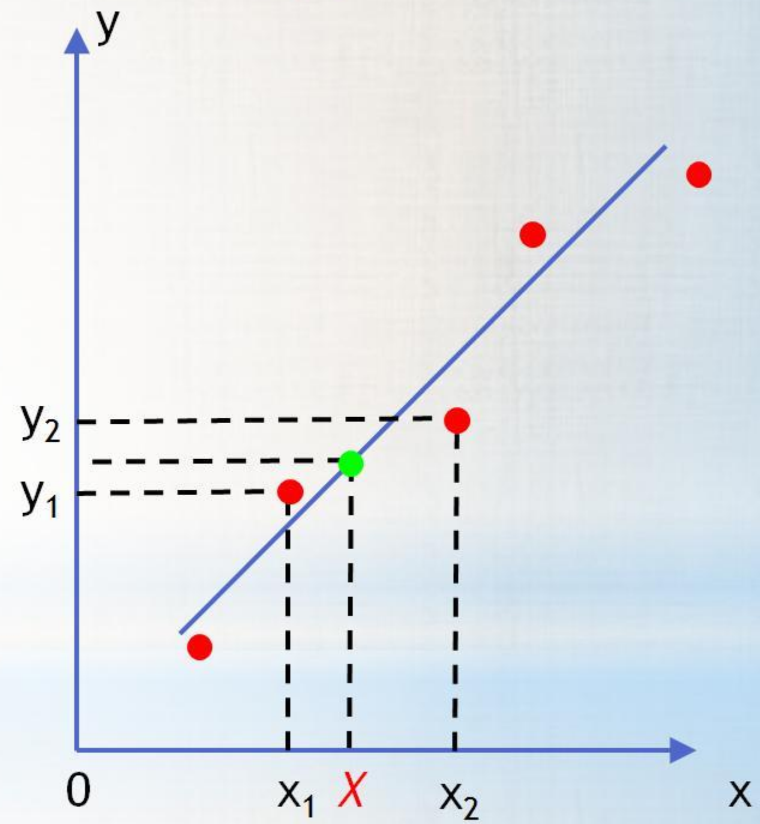
- ***the given data points* are regarded as the approximate ones**

x	x_0	x_1	x_2	...	x_n
$f(x)$	y_0	y_1	y_2	...	y_n

Interpolation



Approximation



Finite differences concept

x	x_0	x_1	x_2	...	x_n
f(x)	y_0	y_1	y_2	...	y_n

↓

$$x_i = x_{i-1} + h$$

$$\Delta y_i = y_{i+1} - y_i$$

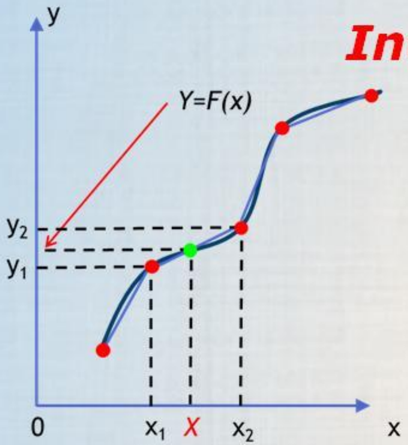
$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$$

$$\Delta^3 y_i = \Delta^2 y_{i+1} - \Delta^2 y_i$$

x	y	Δy	Δ^2y	Δ^3y	Δ^4y
x[0]	y[0]	y[1]-y[0]	Δy[1]-Δy[0]	Δ^2y[1]-Δ^2y[0]	Δ^3y[1]-Δ^3y[0]
x[1]	y[1]	y[2]-y[1]	Δy[2]-Δy[1]	Δ^2y[2]-Δ^2y[1]	
x[2]	y[2]	y[3]-y[2]	Δy[3]-Δy[2]		
x[3]	y[3]	y[4]-y[3]			
x[4]	y[4]				

$$\Delta^k y_i = y_{i+k} - k y_{i+k-1} + \frac{k(k-1)}{2!} y_{i+k-2} - \cdots + (-1)^k y_i$$

Interpolation: Newton's 1st formula



x	x_0	x_1	x_2	…	x_n
$f(x)$	y_0	y_1	y_2	…	y_n

$$x_i = x_{i-1} + h$$

$$f(x) = F(x) \qquad \begin{cases} F(x_0) = y_0 \\ F(x_1) = y_1 \\ \dots \\ F(x_i) = y_i \\ \dots \\ F(x_n) = y_n \end{cases}$$

(*) $$F(x) \rightarrow P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)(x-x_1)\dots(x-x_{n-1})$$

$x = x_0$: $y_0 = P_n(x_0) = a_0$

$x = x_1$: $y_1 = P_n(x_1) = a_0 + a_1(x_1-x_0)$

$x = x_2$: $y_2 = P_n(x_2) = a_0 + a_1(x_2-x_0) + a_2(x_2-x_0)(x_2-x_1)$

$$a_1 = \frac{\Delta y_0}{h}$$

$$a_2 = \frac{\Delta^2 y_0}{2!\,h^2}$$

$$a_3 = \frac{\Delta^3 y_0}{3!\,h^3}$$

(**) $$a_k = \frac{\Delta^k y_0}{k!\,h^k}$$

Interpolation: Newton's 1st formula

(**) $a_k = \frac{\Delta^k y_0}{k!\, h^k}$

x	x_0	x_1	x_2	…	x_n
f(x)	y_0	y_1	y_2	…	y_n

$x_i = x_{i-1} + h$

(*) $F(x) \rightarrow P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$

(**) into (*):

$$P_n(x) = y_0 + \frac{\Delta^1 y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1) + \cdots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Let $\frac{x - x_0}{h} = t$

So called ("aka") interpolation forward

$$P_n(x) = P_n(x_0 + th) = y_0 + t\Delta^1 y_0 + \frac{\Delta^2 y_0}{2!}t(t-1) + \cdots + \frac{\Delta^n y_0}{n!}t(t-1)(t-2)\ldots(t-n+1)$$

@ Home:

1) compare Lagrange and Newton formulas:

2) interpolation backward (2nd Newton formula)

3) what one can do if the nodes are not equally distributed?

Ex. 1:

x	-2	0	2
f(x)	3	0	2

Using for numerical differentiation

x	x_0	x_1	x_2	…	x_n
$f(x)$	y_0	y_1	y_2	…	y_n

$$f'(x) \approx P'_n(x)$$

for any node:

$$f'(x_0) = \frac{1}{h}\left(\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \frac{\Delta^5 y_0}{5} - \cdots\right)$$

Ex. 1: $y = \sqrt{x}$

x	y	Δy	Δ^2y	Δ^3y	Δ^4y
32	5.657	0.088	-0.001	0.000	0.000
33	5.745	0.086	-0.001	0.000	
34	5.831	0.085	-0.001		
35	5.916	0.084			
36	6				

Numerical differentiation

Ex. 2: $y = x^3 - 2\sqrt{x}$

Ex. 3: $y = x^3 - \dfrac{x^4 - 3x^2 - 7}{x^3 + 25}$

Ex. 4: $y = x^2 - \sqrt{x} \cdot \sin(x)$

Ex. 5: $y = x^2 - \left(\sqrt{x} \cdot \sin(x)\right)^x$