- Part II.

Math methods for data analysis:

interpolation and approximation methods

II.1 Interpolation using Lagrange formula

Interpolation ***– a general term (concept) of constructing new data points within the range of a discrete set of known data points***

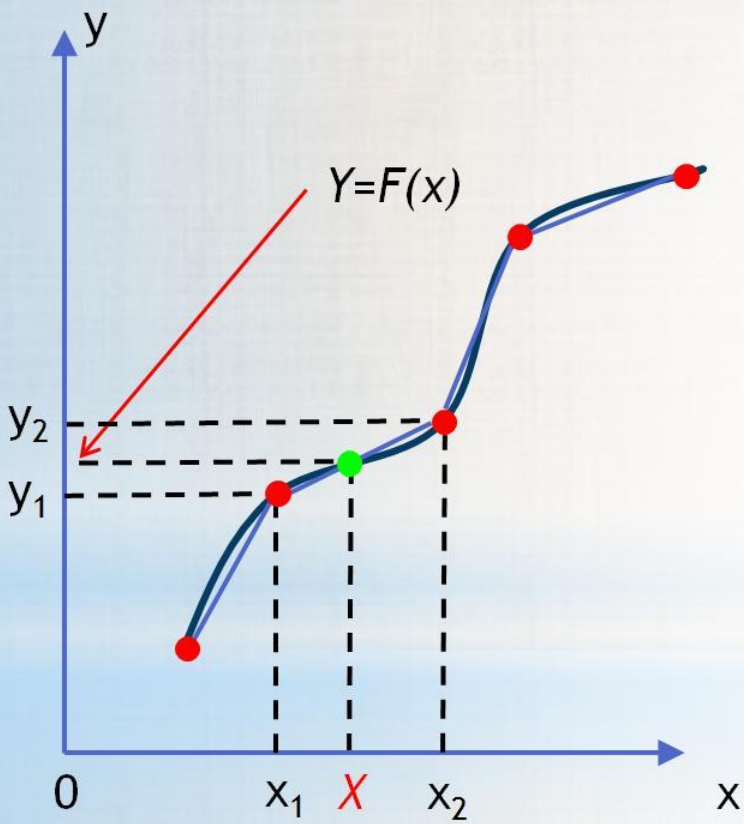
- ***the given data points are considered to be exact nodes of the function to be constructed***

(unlike Interpolation), Approximation – a concept of finding the best possible fit to the given data points, which would allow to predict values outside of the measured range of a discrete set of known data points

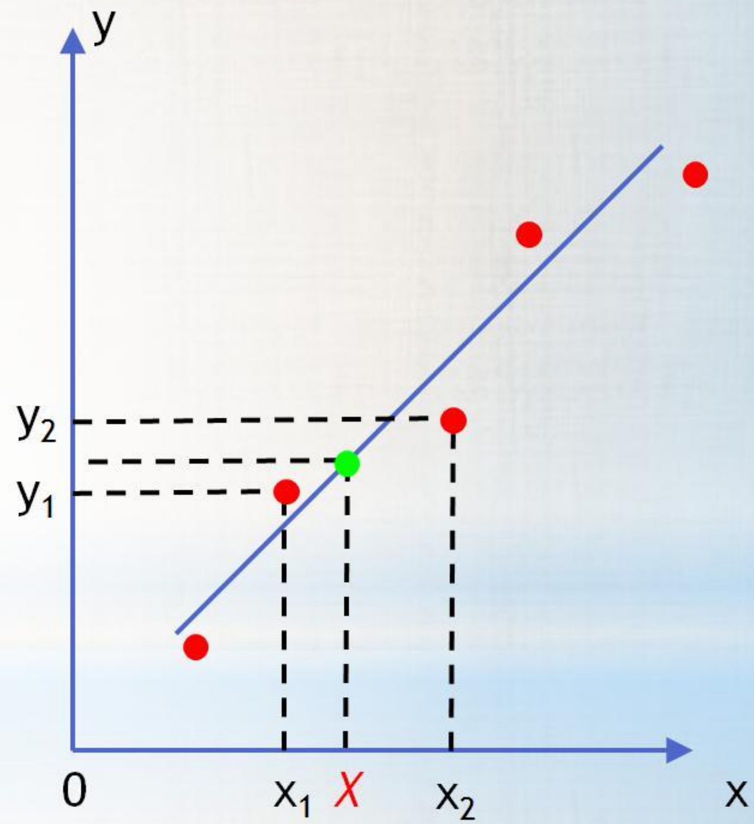
- ***the given data points are regarded as the approximate ones***

x	x_0	x_1	x_2	…	x_n
$f(x)$	y_0	y_1	y_2	…	y_n

Interpolation



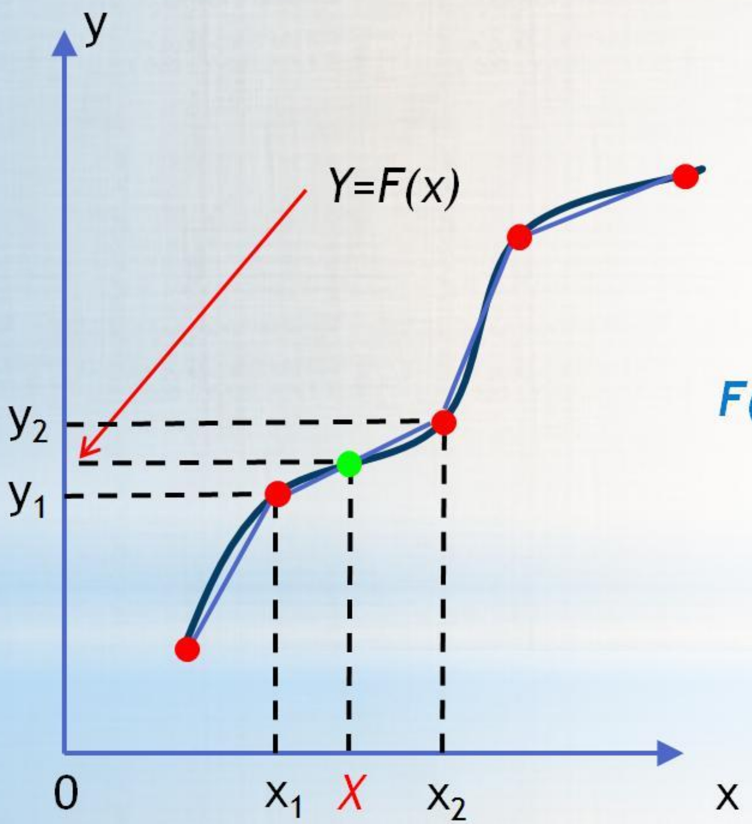
Approximation



Interpolation

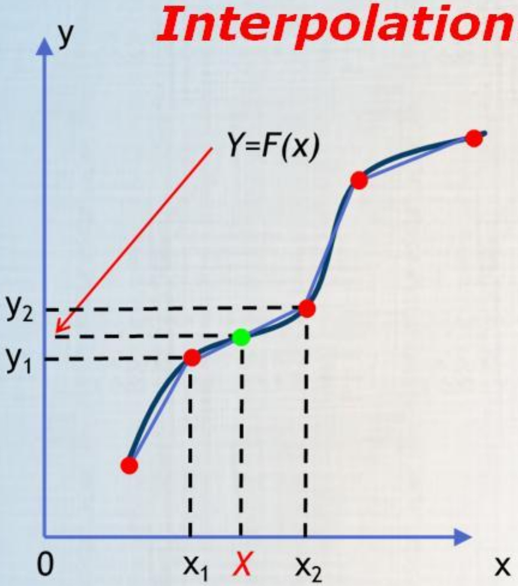
x	x_0	x_1	x_2	…	x_n
$f(x)$	y_0	y_1	y_2	…	y_n

$$f(x) = F(x) \quad \begin{cases} F(x_0) = y_0 \\ F(x_1) = y_1 \\ \dots \\ F(x_i) = y_i \\ \dots \\ F(x_n) = y_n \end{cases}$$



$$F(x) \rightarrow P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a^{n-1}x + a_n$$

Interpolation: Lagrange formula



x	x_0	x_1	x_2	…	x_n
$f(x)$	y_0	y_1	y_2	…	y_n

$$F(x) \rightarrow P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a^{n-1}x + a_n$$

$$L_n(x) = \ell_0(x) + \ell_1(x) + \ell_2(x) + \ldots + \ell_n(x)$$

$$\text{where } \ell_i(x_k) = \begin{cases} y_i, & i=k \\ 0, & i \neq k \end{cases}$$

$$\ell_i(x) = C_i \cdot (x - x_0)(x - x_1)(x - x_2) \cdot\ldots\cdot (x - x_{i-1})(x - x_{i+1}) \cdot\ldots\cdot (x - x_n)$$

$$C_i = \frac{y_i}{(x_i - x_0)(x_i - x_1)(x_i - x_2) \cdot\ldots\cdot (x_i - x_{i-1})(x_i - x_{i+1}) \cdot\ldots\cdot (x_i - x_n)}$$

$$L_n = \sum_{i=0}^{n} y_i \frac{(x - x_0)(x - x_1)(x - x_2) \cdot\ldots\cdot (x - x_{i-1})(x - x_{i+1}) \cdot\ldots\cdot (x - x_n)}{(x_i - x_0)(x_i - x_1)(x_i - x_2) \cdot\ldots\cdot (x_i - x_{i-1})(x_i - x_{i+1}) \cdot\ldots\cdot (x_i - x_n)}$$

Interpolation: Lagrange formula

$$\mathrm{L}_n = \sum_{i=0}^{n} y_i \frac{(x-x_0)(x-x_1)(x-x_2)\cdot\ldots\cdot(x-x_{i-1})(x-x_{i+1})\cdot\ldots\cdot(x-x_n)}{(x_i-x_0)(x_i-x_1)(x_i-x_2)\cdot\ldots\cdot(x_i-x_{i-1})(x_i-x_{i+1})\cdot\ldots\cdot(x_i-x_n)}$$

Ex. 1:

x	1	3	4
f(x)	12	4	6

Numerical term C_i

Functional term

coefficients

$$l_0 = y_0 \cdot \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{y_0}{(x_0-x_1)(x_0-x_2)} \cdot \frac{x^2 + x\cdot(-x_1-x_2) + x_1\cdot x_2}{1}$$

$$l_1 = y_1 \cdot \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{y_1}{(x_1-x_0)(x_1-x_2)} \cdot \frac{x^2 + x\cdot(-x_0-x_2) + x_0\cdot x_2}{1}$$

$$l_2 = y_2 \cdot \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{y_1}{(x_2-x_0)(x_2-x_1)} \cdot \frac{x^2 + x\cdot(-x_0-x_1) + x_0\cdot x_1}{1}$$

$$\mathbf{L}_n = \boldsymbol{l_0} + \boldsymbol{l_1} + \boldsymbol{l_2}$$

Interpolation: Lagrange formula

Ex. 1:

x	1	3	4
f(x)	12	4	6

$$\mathrm{L}_n = \sum_{i=0}^{n} y_i \frac{(x-x_0)(x-x_1)(x-x_2)\cdot\ldots\cdot(x-x_{i-1})(x-x_{i+1})\cdot\ldots\cdot(x-x_n)}{(x_i-x_0)(x_i-x_1)(x_i-x_2)\cdot\ldots\cdot(x_i-x_{i-1})(x_i-x_{i+1})\cdot\ldots\cdot(x_i-x_n)}$$

Numerical term C_i ↓ Functional term ↓

$$l_0 = \frac{y_0}{(x_0-x_1)(x_0-x_2)} \cdot \frac{x^2 + x\cdot(-x_1-x_2) + x_1\cdot x_2}{1}$$

$$l_1 = \frac{y_1}{(x_1-x_0)(x_1-x_2)} \cdot \frac{x^2 + x\cdot(-x_0-x_2) + x_0\cdot x_2}{1}$$

$$l_2 = \frac{y_2}{(x_2-x_0)(x_2-x_1)} \cdot \frac{x^2 + x\cdot(-x_0-x_1) + x_0\cdot x_1}{1}$$

$$\mathbf{L}_n = \boldsymbol{l}_0 + \boldsymbol{l}_1 + \boldsymbol{l}_2$$

Construct a table to calculate all the coefficients needed

F(x) = ax^2 +bx+c				
a	b		c	
1	-x[1]-x[2]	-7	x[1]*x[2]	12
1	-x[0]-x[2]	-5	x[0]*x[2]	4
1	-x[0]-x[1]	-4	x[0]*x[1]	3

C[i]		
2	=	12/[(1-3)(1-5)]
-2		
2		

ℓ_0	2	-14	24
ℓ_1	-2	10	-8
ℓ_2	2	-8	6

L	polinomial coefficients		
	2	-12	22

Interpolation: Lagrange formula

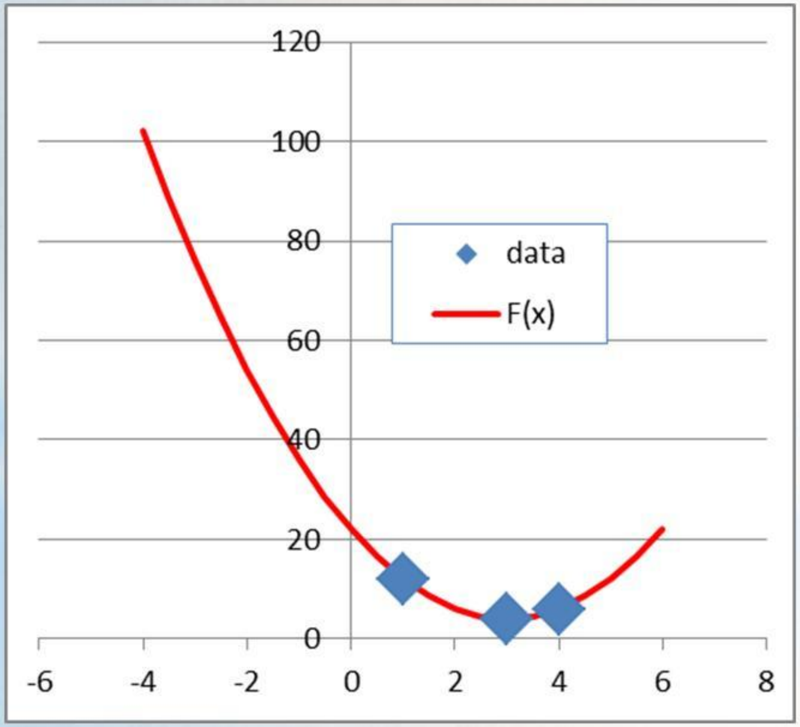
Ex. 1:

x	1	3	4
f(x)	12	4	6

L	polinomial coefficients		
	2	-12	22

Construct plot ***F(x)***, using functional dependence and a step of 0.5

x	F(x)
-4	102
-3.5	88.5
-3	76
-2.5	64.5
-2	54
-1.5	44.5
-1	36
-0.5	28.5
0	22
0.5	16.5
1	12
1.5	8.5
2	6
2.5	4.5
3	4
3.5	4.5
4	6
4.5	8.5
5	12
5.5	16.5
6	22



Interpolation: Lagrange formula Example of the Excel worksheet

i	x[i]	y[i]
0	1	12
1	3	4
2	4	6

C[i]		
2	=	12/[(1-3)(1-5)]
-2		
2		

$F(x) = ax^2 + bx + c$				
a	b		c	
1	-x[1]-x[2]	-7	x[1]*x[2]	12
1	-x[0]-x[2]	-5	x[0]*x[2]	4
1	-x[0]-x[1]	-4	x[0]*x[1]	3

ℓ_0	2	-14	24
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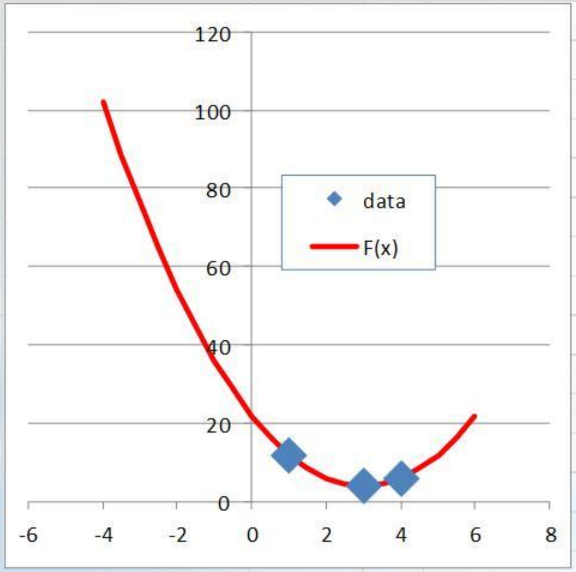

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0	22
0.5	16.5
1	12
1.5	8.5
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3.5	4.5
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5.5	16.5
6	22

Interpolation: Lagrange formula

$$\mathrm{L}_n = \sum_{i=0}^{n} y_i \frac{(x - x_0)(x - x_1)(x - x_2) \cdots (x - x_{i-1})(x - x_{i+1}) \cdots (x - x_n)}{(x_i - x_0)(x_i - x_1)(x_i - x_2) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}$$

Ex. 2:

x	-2	1	2
f(x)	3	0	2

Ex. 3:

x	0.41	1.55	1.91	2.67	3.84
f(x)	2.63	3.75	?	4.87	5.03