Practical problems /use numerical methods/:

1. Find area of a flat figure *D* defined by the curvatures given below and noting that

$$S_D = \iint_D dx\,dy$$

1) $\begin{cases} y = x^2 \\ y = \frac{3-x}{2} \\ y = 0 \end{cases}$
4) $\begin{cases} x = 4 - y^2 \\ x + 2y - 4 = 0 \end{cases}$
7) $\begin{cases} x + y = 2 \\ y = x \\ y = 0 \end{cases}$
10) $\begin{cases} x + y - 5 = 0 \\ xy = 4 \end{cases}$

2) $\begin{cases} x = 2 \\ y = x \\ xy = 1 \end{cases}$
5) $\begin{cases} x = y^2 - 2y \\ x + y = 0 \end{cases}$
8) $\begin{cases} x = 0 \\ y = \pi \\ y = x \end{cases}$
11) $\begin{cases} y^2 = 10x + 25 \\ y^2 + 6x = 9 \end{cases}$

3) $\begin{cases} y = x \\ y = 5x \\ y = 1 \end{cases}$
6) $\begin{cases} x = 4y - y^2 \\ x + y = 6 \\ y = 0 \end{cases}$
9) $\begin{cases} y = \frac{5}{9} \cdot x^2 \\ x = \frac{3}{25} \cdot y^2 \end{cases}$
12) $\begin{cases} y = \frac{\sqrt{x}}{2} \\ y = \frac{1}{2x} \\ x = 16 \end{cases}$

2. Find moment of inertia (with respect to the Cartesian axes) of a flat plate defined by the given curvatures. Note that

$$\begin{cases} J_X = \iint_D y^2 \rho(x,y)\,dx\,dy \\ J_Y = \iint_D x^2 \rho(x,y)\,dx\,dy \end{cases}$$

1) $\begin{cases} x = 2 \\ y = 0 \\ x = 2y^2 \ (y \geq 0) \\ \rho(x,y) = \frac{7x^2}{2} + 6y \end{cases}$
3) $\begin{cases} y = 2x - x^2 \\ y = x \\ \rho(x,y) = x \end{cases}$
5) $\begin{cases} y = 2x^3 \\ y^2 = 2x \\ \rho(x,y) = y^2 \end{cases}$
7) $\begin{cases} \frac{x^2}{4} + y^2 = 1 \\ \rho(x,y) = 4y^2 \end{cases}$

2) $\begin{cases} y = x^2 \\ x = y^2 \\ \rho(x,y) = x \end{cases}$
4) $\begin{cases} x = 0 \\ x = 2 \\ y = 0 \\ y = 3 \\ \rho(x,y) = x^3 y^2 \end{cases}$
6) $\begin{cases} x = y^2 \\ y = x^2 \\ \rho(x,y) = y \end{cases}$
8) $\begin{cases} x^2 + y^2 = 1 \\ x^2 + y^2 = 4 \\ \rho(x,y) = \frac{x+y}{x^2+y^2} \end{cases}$

3. Find coordinates of the centre of the mass of a flat plate ($x_c = M_Y/m$, $y_c = M_X/m$) using

$$\begin{cases} M_X = \iint\limits_D y\rho(x,y)\,dx\,dy \\ M_Y = \iint\limits_D x\rho(x,y)\,dx\,dy \\ m = \iint\limits_D \rho(x,y)\,dx\,dy \end{cases}$$

1) $\begin{cases} y = \sqrt{x} \\ y = 2 - x \\ x = 0 \\ \rho(x,y) = y \end{cases}$

3) $\begin{cases} y = 2\sqrt{x} \\ x + y = 3 \\ y = 0 \\ \rho(x,y) = 1/x \end{cases}$

5) $\begin{cases} y = x^2 \\ y = 2 - x \\ \rho(x,y) = y^2 \end{cases}$

7) $\begin{cases} x^2 + y^2 = 2x \\ \rho(x,y) = 4y^2 \end{cases}$

2) $\begin{cases} y = 4 - x^2 \\ y = 0 \\ \rho(x,y) = x \end{cases}$

4) $\begin{cases} y = x^3 \\ y = -\sqrt{x} \\ x = 2 \\ \rho(x,y) = x^3 \end{cases}$

6) $\begin{cases} y = 2x - x^2 \\ y = 2 - x^2 \\ x = 0 \\ \rho(x,y) = y \end{cases}$

8) $\begin{cases} x^2 + y^2 = 2x \\ x^2 + y^2 = 6x \\ \rho(x,y) = \frac{x^2+y^2}{x+y} \end{cases}$

4. Find the moment of inertia of a homogeneous ball, with mass *m*.

5. Find the moment of inertia of an inhomogeneous ball, with density ρ:

1) $\rho = \rho_0 \cdot e^{-r}$

2) $\rho = \rho_0 \cdot e^{-\frac{r^2}{2}+2r}$

6. Find the charge flown through a circuit during T = 1, 10, 100 min if

1) $I = I_0 \cdot e^{-\alpha \cdot \frac{t}{T}}$

2) $\begin{cases} U = U_0 \cdot e^{-\alpha \cdot \frac{t}{T}} \\ R = R_0 \cdot \frac{\sqrt{T}}{\sqrt{t}} \end{cases}$

Vary parameters I_0, U_0, R_0, α within a meaningful range.