STUDY OF THE TEMPERATURE DEPENDENCE OF THE ELECTRICAL CONDUCTIVITY OF METALS AND SEMICONDUCTORS

1 Introduction

The electrical conductivity of a metal σ can be expressed as

$$\sigma = e\, n_e \mu_e, \qquad (1)$$

where e is the electron charge, n_e is the electron concentration, μ_e is the electron mobility, which is numerically equal to the electron drift velocity in an electric field with a unit intensity.

For semiconductors, where, in addition to electrons, charge transfer is carried out by mobile positive charge carriers — holes, an extra term, related to holes, should be added to the formula (1) :

$$\sigma = e\, n_e \mu_e + e\, n_p \mu_p, \qquad (2)$$

where μ_p and n_p are the mobility and holes concentration, respectively.

It can be seen from relations (1) and (2), that the temperature dependence of the electrical conductivity of various substances is determined by the temperature dependence of the mobility and concentration of free charge carriers.

The classical theory of electrical conductivity falls to explain the dependence σ on temperature *T,* which is observed in experiment. Within the framework of the classical theory, it is also unclear why some crystals are good conductors, others are dielectrics or semiconductors. The band theory of solids gave answers to these and many other questions.

When atoms combine into a crystalline solid, the structure of the energy levels of electrons undergoes important changes. These changes almost do not affect the levels of the inner, filled shells. The wave functions of the outer electrons substantially overlap, which leads to the consolidation of these electrons: they belong not to individual atoms, but to the whole crystal. The energy levels of the external electrons are split, forming allowed energy bands, separated by forbidden bands. The width of the bands is determined by the bond between the atoms in the crystal and does not depend on the number of atoms in the crystal, and the number of levels in the band is equal to the number of atoms. Thus, the distance between the levels is so small that it does not make sense to talk about the position of individual levels in the zone. The states of consolidated electrons located at different levels of the same band differ in momentum (or, more precisely, in quasi-momentum), and, therefore, in direction and speed of motion.

The conductivity of crystals is determined by the level distribution of electrons. In *insulators*, electrons concentrate in the lowest energy band (valence band). The next allowed zone (conduction band) contains no electrons. The width of the band gap is large compared to the characteristic thermal energy kT, so the electrons in the valence band under normal conditions are not able to "jump over" the band gap. Because of the symmetry of the crystal, the number of electrons in the valence band and moving in opposite directions is the same - there is no electric current. In the presence of an electric field, electrons cannot change their motion, since there are no free energy states in the zone.

In *metals*, electrons only partially occupy the last of the occupied zones. In the presence of an electrical field, electrons can occupy free states, corresponding to obtaining a "counter field" pulse, and the crystal conducts the current.

Semiconductors include substances that are insulators at low temperatures. The width of the band gap ΔE in semiconductors is small; therefore, at ordinary temperatures, thermal motion transfers part of the electrons from the valence band to the conduction band. The electrical conductivity associated with the presence of electrons in the conduction band is called electron, and because of the appearance of free states in the valence band, the so-called hole conduction occurs.

Let's consider the temperature dependence of the concentration of free charge carriers n in metals and semiconductors.

In metals, the concentration of free electrons is high (of the order of 10^{28} - 10^{29} m^{-3}) and does not vary with temperature.

In pure semiconductors, on the contrary, the concentration of free charge carriers strongly depends on temperature. At $T = 0$, the valence band is completely filled with electrons, and the conduction band is empty. If the temperature of the crystal is increased, then in equal quantities there will appear conduction electrons and holes, the concentration of which in a pure semiconductor is determined by the probability of such a transition:

$$n_e = n_p \approx n_0 \exp\left(-\frac{\Delta E}{2kT}\right), \qquad (3)$$

where k is the Boltzmann constant, n_0 is a constant. In this case, one speaks about the intrinsic conductivity of the semiconductor.

It can be seen from the expression (3), that the concentration electrons and holes n_e and n_p tends to zero as the temperature decreases. However, the carrier concentration does not drop to zero, because in any semiconductor there are impurities that are provides extra electrons (in the n-type conductivity material) or holes (in the p-type material). Since the ionization energy E_i of impurities is hundreds of times smaller than the band gap ΔE, then at temperatures, when $n_e = n_p \to 0$ the impurities are completely ionized. This so-called "impurity depletion region" is a temperature range in which the concentration of charge carriers produced by ionization of impurities does not change.
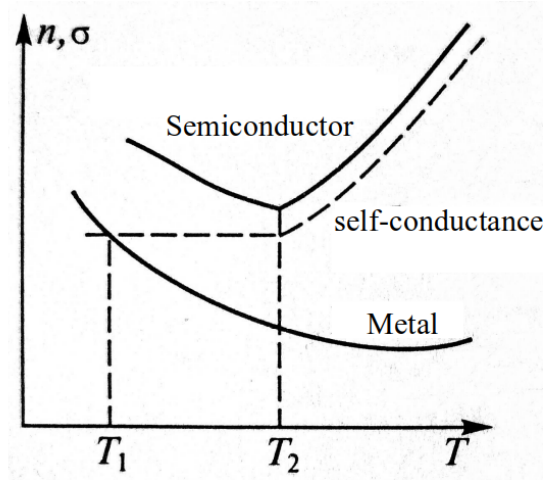
Fig.1. The dashed curve shows the temperature dependence of the concentration of conduction electrons in semiconductors. The temperature range from T_2 to T_1 corresponds to the depletion of impurities $kT_1 \geq E_i$.

The *mobility* of charge carriers in crystalline solids is associated with their scattering on defects (deviations from ideality) of the crystal lattice. We consider one type of such defects: thermal oscillations of ions around the equilibrium position at $T > 0$. During thermal vibrational motion, an ion periodically shifts from an equilibrium position, causing scattering of charge carriers. The higher the average vibration energy ($\sim kT$), the greater the amplitude of atomic vibrations in the lattice sites and the lower the mobility, i.e. with increasing of temperature, mobility decreases. A rigorous calculation shows that

$$\mu \sim T^{-\alpha}, \qquad (4)$$

For metals $\alpha \approx 1$, while for semiconductors $\alpha \approx 3/2$.

Analyzing the temperature dependence of concentration and mobility, one can make some preliminary conclusions about the temperature dependence of the conductivity of metals and semiconductors.

In *metals*, the carrier concentration is unchanged and therefore the temperature dependence of the conductivity is determined only by the electron mobility. With temperature increasing, the conductivity of metals decreases, and the resistance increases. The dependence of the specific resistance ρ on temperature is characterized by the temperature coefficient of resistance of a given metal:

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}.$$

Generally speaking, the temperature coefficient of resistance of a given metal is different at different temperatures. However, the change of α with temperature is not very large, so in a relatively narrow range of temperatures, it can be considered constant, while ρ dependence on temperature is linear:

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$$\rho = \rho_0(1 + \alpha t), \qquad (5)$$

where ρ_0 is the specific resistance at $0\ °C$, t is the temperature on the Celsius scale.

In *semiconductors*, the concentration and mobility of free charge carriers also vary with temperature. At sufficiently low temperatures, in the area of depletion of impurities ($n = constant$), the electrical conductivity $\sigma(T)$ is proportional to $\mu(T)$, therefore, as in the case of metals, with increasing of temperature, the conductivity decreases, and the resistance increases.

At sufficiently high temperatures ($T > T_2$) in the self-conductivity region

$$\sigma = e\,n_e\,\mu_e + e\,n_p\,\mu_p = e(\mu_e + \mu_p)\,n_0 \exp\left(-\frac{\Delta E}{2\,kT}\right); \qquad (6)$$

since μ_e and μ_p are weakly (in comparison with the exponent) depend on temperature, (6) can be written approximately as:

$$\sigma = \sigma_0 \exp\left(-\frac{\Delta E}{2\,kT}\right), \qquad (7)$$

where $\sigma = e(\mu_e + \mu_p)n_0$.

The solid line on Fig. 1. stands for the temperature dependence of conductivity of semiconductors and metals.

The band gap of a semiconductor can be determined from the dependence of its resistance R on temperature. Since $R = l/(\sigma S)$, where l is the length of the sample, S is the area of its cross section, then taking into account equation (7) under the condition of self-conductance region

$$R = R_0 \exp\left(\frac{\Delta E}{2\,kT}\right),$$

or

$$\ln R = \ln R_0 + \frac{\Delta E}{2\,kT}$$

In this practical, the study of the semiconductor resistance is performed at $T \geq T_2$. Depending on the purity of the sample and the temperature, the impurity conductivity may have higher or lower contribution to the total one. It distorts the temperature dependence of semiconductor self-conductivity. In order to determine the width of the band gap correctly, it is necessary to make measurements in a wide range of temperatures and select a site where the dependence of resistance on $1/T$ has an exponential law.

To determine the width of the band gap, the dependence *ln R* on *1/T* is plotted. According to the formula (8), in the region of self-conductivity the graph should have the form of a straight line, with the slope equals to $\frac{\Delta E}{2\,k}$, which gives

$$\Delta E = 2\,k \tan \alpha. \qquad (9)$$

2 Experimental setup

The setup (see Fig.2) allows you to measure the resistance of the samples and their temperature in the process of heating and cooling.

Fig.2. The picture of the setup.

Several elements of the setup are installed inside the case 1: an electric heater with studied samples placed inside, a temperature sensor (a resistance thermometer), a fan for cooling the samples, and power sources for the electric heater and the fan. The electric heater serves to heat the samples to a maximum temperature of *120* °C for *15 - 20* minutes. The fan serves to accelerate the cooling process of the samples. Several switches and displays are located on the front panel of the case 2.

The positions of the "ОБРАЗЕЦ" switch correspond to the connection of the following samples:

"0" - the measurement input of the device is shorted

"1" - a metal (copper);

"2" - an alloy with a low temperature coefficient of resistance (manganin or constantan);

"3" - semiconductor (semiconductor thermistor).

On the front panel of the case 2 there are buttons "НАГРЕВ" and "ВЕНТ", which can switch on and off (by second push) the electric heater and fan. Displays "ОМ" and "°С" record the values of resistance and temperature of the samples during the measurements.

Using the "STOP IND" button, one can pause the readings on the displays of temperature and resistance. When the pause mode is on, the readings on these displays are fixed and identical to those at moment of activation. The operation mode of the setup does not affected by the pause mode, in other words, the heating or cooling process of the sample will continue. The second push will lead to indication of the new values of the measured parameters, which associated with the current physical parameters of the sample.

3 Measurement and data processing

Connect the power cords to the cases 1 and 2 and switch on the setup using the switches "СЕТЬ" on the rear panel of the case 1 and the front panel of the case 2. Allow the units to warm up for 5 minutes.

Task 1. Measurement of the temperature coefficient of resistance of the metal.

Set the "ОБРАЗЕЦ" switch to the position "1", which is associated with the metal.

Press the "НАГРЕВ" button on the front panel of the case 2. Watching the temperature (0 °C) and resistance displays, record the dependence $R(t)$ every 5 °C in the range from room temperature upto 120 °C. (If temperature reaches 125 °C, an overheat protection will activate: the heater automatically switches off while the fan switches on.) When the maximum temperature reaches 120 °C, switch off the heater by pressing the "НАГРЕВ" button and switch on the fan with the "ВЕНТ" button to cool the sample. Keep tracking the displays reading and record the $R(t)$ dependence, while the sample is cooling. The results of measurements should be recorded in the protocol in the tables.

A small discrepancy between the resistance values at the same temperatures when measured under the heating and cooling modes is related to the different thermal inertia of the temperature sensor and the samples.

Plot on the same graph dependencies $R(t)$ obtained under heating and cooling measurement. Extrapolate the data to determine the resistance R_0 at 0 °C. Calculate the temperature coefficient of resistance of the metal from the slope of the graphs and the R_0 value.

Estimate the errors of measured result.

Task 2. Calculating the band gap of a semiconductor.

Set the "ОБРАЗЕЦ" switch to position "3" - semiconductor.

Repeat the measurements with this sample, as described in the task 1.

At the end of the measurements, set the "ОБРАЗЕЦ" switch to the "0" position

Turn off the power supply with the "СЕТЬ" switch.

Plot the $R(t)$ dependencies on the same graph.

Plot the $\ln R = f(1/T)$; from the slope of its linear part (at relatively high temperatures), calculate the width ΔE of the band gap of the semiconductor under study in electron-volts.

Estimate the errors of measured result.

4 Questions

1. What is the difference in the energy spectrum of electronic states in semiconductors and dielectrics?

2. What is the temperature dependence of the mobility of free charge carriers in metals and semiconductors?

3. How does the concentration of current carriers depend on temperature in metals and in semiconductors - chemically pure and with impurities?

4. What is the temperature dependence of the resistivity of metals and semiconductors?

5. How is the growth of specific resistance of a metal explained from the standpoint of classical physics and from the standpoint of quantum physics?

6. What is the statistical distribution, which describes the electron gas in the metal? What does the Fermi temperature depend on?

7. Calculate the Fermi energy and Fermi temperature for copper. Take the number of free electrons equal to the number of atoms. Copper density $\rho = 8.94 \times 10^3$ kg/m^3.

8. What is a degenerate electron gas? Is electron gas degenerated in metal at room temperature? In a semiconductor?