PRACTICAL 1.12

RESONANCE MEASUREMENT OF THE NATURAL FREQUENCIES OF A STRING

Objective: determination of natural frequencies of transverse vibrations of a string and calculation of the mass density of the material of the string.

Equipment: string, vibrator, sound generator, weights.

INTRODUCTION

The vibrational motion of a tense uniform string can be represented as a superposition of normal (natural) modes with various frequencies. A stationary picture is observed if the vibrations are driven by an external periodic (in particular, harmonic) force orthogonal to the string. The displacement of the particles of the string in this case is perpendicular to the string. The external perturbation causes the propagation of a transverse wave along the string, which under certain conditions leads to a stranding wave. This is called the resonance condition.

If the ends of the string are fixed, the displacement at the endpoints is zero, and the transverse standing waves are only possible provided

$$l = n\frac{\lambda_n}{2} \text{ or } \nu_n = \frac{ns}{2l}, \tag{1}$$

where l is the length of the string, λ_n and ν_n are the wavelength and the frequency, respectively, s is the speed of the transverse wave along the string, and n is a positive coefficient corresponding to the harmonic of the standing wave pattern. Literally, n is the number of the antinodes observed on the string.

The speed of the transverse wave along the string does not depend on the frequency and only depends on the material density ρ and the stress $\sigma = F/S$, F being the tension and S the cross-section area of the string:

$$s = \sqrt{\frac{F}{S\rho}}. \tag{2}$$

Combining Eqs. (1) and (2) produces

$$\nu_n = \frac{n}{2l}\sqrt{\frac{F}{S\rho}} \tag{3}$$

Eq. (3) enables experimental determination of the material density of the string through the direct measurement of ν_n, n, F, and S. In this practical the natural frequencies of the string are measured with the resonance technique.

EXPERIMENTAL SETUP

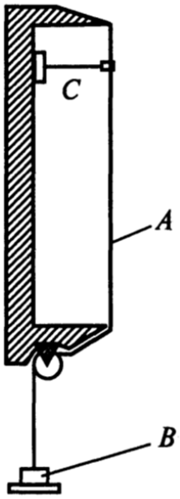

Fig. 12.1 1

A schematic of the experimental setup is shown in Fig. 12.1. The tension of the string A is controlled with weights B. Forced oscillations are driven by the vibrator C, with the frequency of the vibrations set by the sound generator. The amplitude of the forced oscillations is maximal on resonance, *i.e.* when the frequency of the drive is equal to one of the natural frequencies of the string.

MEASUREMENT AND DATA PROCESSING

Task 1. Measure the length l and the diameter d of the string, then measure the natural frequencies of the string for $n = 1, 2, 3, 4$ at $F = 10, 15, 20, 25$ N. Plot $(\nu_n/n)^2$ *vs.* F (use different point-style in the same plot for different n). Using your plot, calculate the material density ρ of the string; make a table of the measured values of F, ν, l, d, n and the calculated values of $(\nu_n/n)^2$, S, ρ.

Task 2. Calculate the speed of the transverse waves along the string at different values of the tension and make a plot of s^2 *vs.* F. Calculate the relative error of the measurement of ρ and s in the experiment.

QUESTIONS AND EXERCISES

1. Derive the equation of standing waves in a string.

2. Draw the configuration of the string when a standing wave is excited in it at $n = 2$ for two time points separated by $T/4$ (T being the period of oscillations) and mark the positions of the zeros and maxima of the velocity, strain, kinetic and potential energies.

3. How different are the oscillation amplitudes of the points on the string for different n at the same tension? Why?

4. Why is it reasonable to plot $(\nu_n/n)^2$ *vs.* F?

5. Why is it important to place the vibrator driving the string as close as possible to its fixed end?

6. Is it systematic uncertainties or random uncertainties that determine the accuracy of your measurements?

PRACTICAL 1.13

RESONANCE MEASUREMENT OF THE OSCILLATION FREQUENCY OF A TUNING FORK

Objective: study resonance phenomena in acoustics and measure the oscillation frequency of the tuning fork.

Equipment: glass tube, metal vessel, rubber hose, two tuning forks, rubber hammer, headphones, ruler.

INTRODUCTION

In a pipe of length l closed at one end, one can excite a standing wave with a maximum of displacement and velocity of the air particles at the open end and zero of these quantities at the closed end. In this case one speaks of the acoustic resonance. The wavelengths of such sound waves λ_n are determined from the relation

$$l = (2n+1)\frac{\lambda_n}{4}, n = 0,1,2,\ldots \quad (1)$$

The corresponding frequencies are given by

$$\nu_n = \frac{s}{\lambda_n} = \frac{s(2n+1)}{4l}, \quad (2)$$

where s is the sound speed.

In this practical, the phenomenon of the acoustic resonance is used to determine the oscillation frequency of a tuning fork.

EXPERIMENTAL SETUP

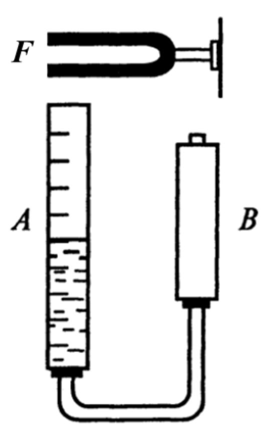

Fig. 13.1

A schematic of the experimental setup is shown in Fig. 13.1. The sound waves excited by the tuning fork F propagate through the air. The glass tube A is connected to the vessel B with a rubber hose. If the water level in the tube is such that the length of the air column satisfies the condition of Eq. (1), then the sound wave will be on resonance, with a maximum of the displacement at the open end of the tube. One can vary the water level in the tube by moving the vessel B up and down the guides.

MEASUREMENT AND DATA PROCESSING

Having excited free oscillations of the tuning fork (with the rubber hammer) and gradually lowering the water level in the tube (starting with the level almost at the open end of the tube), note the water levels corresponding to the maximum sound (acoustic resonance). A pair of headphones connected to the upper end of the tube with a rubber hose can be used to mark the positions corresponding to the maximum sound with a better accuracy.

To improve the accuracy of the measurement, mark three successive positions of the water level when the tuning fork is on resonance with the air column in the tube, corresponding to condition (1). The wavelength of the sound wave can then be deduced as

$$\lambda_{res} = 2(l_3 - l_2), \nu_{res} = \nu_{fork} = \frac{s}{\lambda_{res}}, \tag{3}$$

where l_2 and l_3 are the second resonance levels measured with respect to the open end of the tube.

Task. Make the necessary measurements with two tuning forks and compute their oscillation frequencies. Make a table of your measurement and calculation data (l, λ, ν).

QUESTIONS AND EXCERCISES

1. Write down the equation of the standing wave in the air column in the tube. Is the displacement of the air particles at the maximum or zero at the air-water interface?

2. Draw the displacement of the air particles as a function of time in a standing wave in the tube at n = 2 and mark the positions of the zeros and maxima of the velocity of the air particles, air pressure, and the kinetic and potential energies of the air particles.

3. Is the energy transferred in a standing wave? Explain your answer.

4. Draw the displacement and velocity of points, and the pressure as functions of the coordinate along the tube for two time points separated by $T/4$ (T being the period of oscillations).

5. What is the motion of various parts of the tuning fork after it has been hit with the rubber hammer?