PRACTICAL 1.10

STUDY OF FREE OSCILLATIONS OF A COMPOUND PENDULUM

Objective: To determine the quality factor of a physical pendulum.

Tools and equipment: physical pendulum, measuring scale, lighting bulb with an optical system, plate, stopwatch.

INTRODUCTION

Free oscillations of a real pendulum are damped, since the energy transferred to the system is gradually consumed to overcome friction. The law of decrease of the amplitude depends on nature of the frictional forces acting on the pendulum.

In the experiment, the resistance force and therefore braking torque M_{fr} can be considered proportional to the velocity, i.e., $M_{fr} = -r\frac{d\varphi}{dt}$, where r is the friction coefficient, φ is the angular displacement of the pendulum.

The equation of motion for small oscillations of a pendulum in this case can be written as follows

$$\frac{d^2\varphi}{dt^2} + 2\beta\frac{d\varphi}{dt} + \omega_0^2\varphi = 0, \qquad (1)$$

where $\beta = \frac{r}{2I_z}$ is the damping coefficient, $\omega_0 = \sqrt{\frac{mgl}{I_z}}$ is the intrinsic angular frequency of oscillation of the pendulum, m is the pendulum's mass, I_z is the moment of inertia of the pendulum with respect to the axis of rotation, l is the distance between the center of mass of the pendulum and the axis of rotation.

The solution of equation (1) for the case of $\beta^2 < \omega_0^2$ (small resistance of medium) is of the form

$$\varphi = \varphi_m\cos(\omega t + \Theta), \quad \varphi_m = \varphi_{m_0}e^{-\beta t}, \qquad (2)$$

where $\omega = \sqrt{\omega_0^2 - \beta^2}$ is the angular frequency of free oscillations, φ_{m_0} and Θ are the initial amplitude and phase of the oscillations, respectively.

In addition to the damping coefficient β, a term the quality factor of a system is widely used for analysis of an oscillatory system. The quality factor characterizes a relative decrease of energy of the oscillation over the period:

$$Q = 2\pi \frac{E}{A_{\text{fr}}} = \frac{\omega_0^2}{2\beta\omega}, \qquad (3)$$

where E is the total reserve of energy of the system, A_{fr} is the work against the friction forces over the period of oscillation. For small oscillations with a weak friction ($\beta << \omega_0$), and $\omega \approx \omega_0$ consequently the expression for the quality factor becomes

$$Q = \frac{\omega}{2\beta} \approx \frac{\omega_0}{2\beta}. \qquad (4)$$

The work is dedicated to evaluation of the quality factor of a compound pendulum. As follows from (4), to calculate the quality factor one has to determine the damping coefficient β and the angular frequency of free oscillations of the pendulum.

EXPERIMENTAL SETUP

The pendulum is basically a rod with a massive body mounted at the lower end of it suspended by means of two needle bearings on a special bracket. The pendulum can oscillate in a vertical plane. Mirror located at the upper end of the pendulum reflects incident light from the illuminator ("glint") towards the scale. The "glint" moves along the scale correspondingly to oscillation of the pendulum: the distance d of displacement of the "glint" from the initial position on the scale is equal to $d(t)=L\varphi(t)$, where L is the distance from the mirror to the scale, $\varphi(t)$ is the angular displacement of the pendulum.

To increase the damping a metal plate is attached to the pendulum's rod. Positioning the plate at different angles with respect to the oscillation plane of the pendulum one can change the damping coefficient and the quality factor of the system by several times.

MEASUREMENT AND DATA PROCESSING

To determine the angular frequency of free oscillations of the pendulum the time of 20 complete oscillations is measured. Initial displacement of the "glint" is recommended to choose equal to 12 — 15 divisions of the scale.

The damping coefficient β is determined from a plot of dependence of the oscillation amplitude on time. Initial displacement of the pendulum is 17 divisions of the scale. A stopwatch is turned on, when the oscillation amplitude becomes equal to 15 divisions of the scale. The stopwatch's readings are logged for the amplitude values of 13, 11, 9, 5 and 3 divisions of the scale. The dependencies $\varphi_m(t)$ and $\ln\varphi_m(t)$ are plotted out based on the data obtained.

Task. **Evaluating parameters of the damped pendulum**

Conduct all the measurements required and plot graphs of the dependencies $\varphi_m(t)$ and $\ln\varphi_m(t)$ at two different positions of the stopping plate on the pendulum's rod (the plate needs to be oriented along the oscillation plane of the pendulum and perpendicular to it). For each case calculate ω, β, Q. Write down results of the measurements and calculations and fill a table.

QUESTIONS AND EXERCISES

1. What is called a free oscillation of a pendulum? How are displacement and velocity of a particle shifted in phase in case of free oscillations?

2. What is the damping coefficient, logarithmic decrement? Explain their physical meaning.

3. Does the frequency of intrinsic oscillations of a body depend on its mass?

4. What is the accuracy of evaluation of the intrinsic frequency of oscillations of a pendulum in each experiment?

5. Specify pros and cons of the mirror-based method of measurement used in the work.

6. Is it possible to verify linearity of the dependence of the friction torque on the angular velocity of the pendulum based on the experimental outcome?