PRACTICAL 1.9

MEASUREMENT OF THE FREE-FALL ACCELERATION

WITH ASSISTANCE OF A COMPOUND PENDULUM

Objective: study of the properties of a compound pendulum and experimental evaluation of the free-fall acceleration.

Tools and equipment: compound pendulum, stopwatch, metallic metric ruler (1 meter long).

INTRODUCTION

In case of small deviations from a point of the equilibrium the compound pendulum oscillates with a period *T* given by

$$T = 2\pi\sqrt{\frac{I}{mga}} = 2\pi\sqrt{\frac{I_0 + ma^2}{mga}} \quad , \qquad (1),$$

where *I* is the pendulum's moment of inertia with respect to the oscillation axis, I_0 is the pendulum's moment of inertia with respect to the axis drawn through the center of mass of the pendulum, *a* is the distance between the point of suspension and the center of mass of the pendulum, *m* is the pendulum's mass. The right-hand part of the equation is derived with use of the parallel axis theorem (*aka* Steiner theorem *or* Huygens–Steiner theorem), $I = I_0 + ma^2$, *a* is the distance between the oscillation axis center and the center of mass of the pendulum.

When the amplitude of the oscillation is small, the oscillation could be treated as the harmonic oscillation with the frequency ω_0, and period *T* defined as:

$$T = \frac{2\pi}{\omega_0} = 2\pi \cdot \sqrt{\frac{I}{mga}} = 2\pi \cdot \sqrt{\frac{l_{eq}}{g}} \quad , \qquad (2)$$

here, l_{eq} is the equivalent length of a compound pendulum, $l_{eq} = I/ma$.

With use of the parallel axis theorem, $l_{eq} = a + I_0/(ma)$.

l_{eq} could be determined with use of the graphical method. Figure 9.1 provides a plot of typical dependence of the oscillation period *T* on the distance *x* between the point of suspension and the center of mass of the pendulum. As one can notice, $l_{eq}=a_1+a_2$ in case of equal periods, when $a=a_1$ and $a=a_2$.

MEASUREMENT AND DATA PROCESSING

Task 1. **Evaluation of *g* based on a graph of dependence of the oscillation period of the physical pendulum on the position of the point of suspension.**

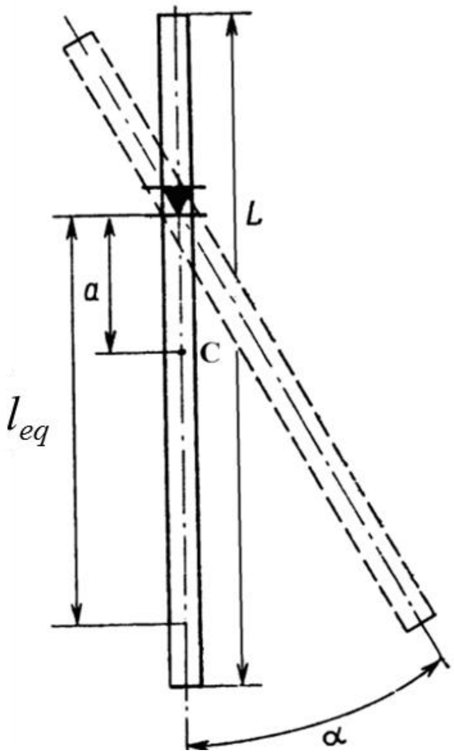

Fig. 9 1

The uniform rod of length L is used as the pendulum. The oscillation period of the pendulum is measured with respect to a position of the point of suspension. The position is gradually tuned from the middle (i.e. the center of mass) to the end of the rod during the experiment (a set of 15 positions or more has to be implemented). First, with use of a prism, determine the center of mass of the pendulum. Second, position the suspension point at 2 cm above the center of the mass, and measure time needed for 40 full oscillations. Repeat this measurement 3-4 times. (Alternatively, position the prism at the end of the pendulum and move it towards the center of the mass with a step of ~ 5 cm aside and ~1 cm closer to the center).

Repeat the measurement for different suspension points. Use a small step for *a* around the minima of the oscillation period (see Fig. 9.2.).

The highest precision is required for measurements of the oscillation period of small value. Based on the experimental outcome, the curve of dependence of the oscillation period on the distance between the point of suspension and the center of mass of the rod is plotted (the change of position of the pendulum's center of mass during motion of the knife-edge along the rod is neglected due to the smallness of the knife-edge's mass compared to that of the rod).

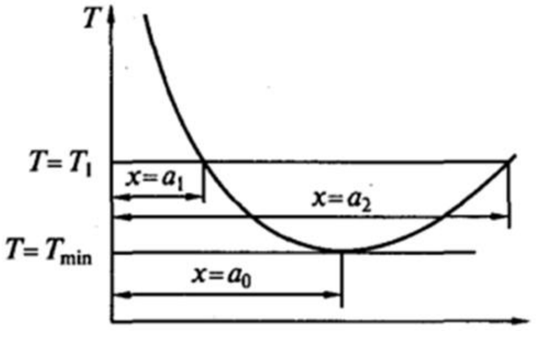

Fig. 9 2

The plot is further used for evaluation of the equivalent lengths ($l_{eq}=a_1+a_2$) corresponding to three different values of the period. Subsequently, the free-fall acceleration is evaluated for each case with use of (2), and the final result is calculated as a mean of these three values of ***g*.**

Conduct all the measurements required, plot out the curve of dependence $T=f(a)$ and evaluate the free-fall acceleration using the rod-shaped pendulum. Find out the theoretical ***g*** considering the pendulum as a thin solid homogeneous cylinder and using *T(a)* data. Compare the experimental and theoretical approaches.

Write down the results of measurements and calculations *t, T, a, l, g* and fill a table. Estimate an experimental error of the evaluation of *g*.

QUESTIONS AND EXERCISES

1. Provide definition of a physical pendulum and evaluate the equation (1).

2. How can one evaluate position of the center of mass of a physical pendulum, if positions of the conjugate points of suspension corresponding to the minimum oscillation period are provided?

3. Why does the amplitude of the pendulum's oscillation need to stay small during measurement of its period.

4. What is the main source of errors contributing to the evaluation of *g* in case of methods employed in the work?

5. How does the friction presented in the system impact on precision of the evaluation of *g*?