PRACTICAL 9

Uncertainty relation for photons

Objective: experimental confirmation of the uncertainty relation for photons.

Experimental equipment: light source - laser, variable-width slit, optical bench, screen, ruler, microscope.

INTRODUCTION

The uncertainty relation is one of the fundamental principles of modern physics. For non-relativistic particles, it can be formulated as follows: the uncertainty of the position of the particle Δx and the uncertainty of the projection of its momentum in the same direction Δp_x should satisfy the relation

$$\Delta x \cdot \Delta p_x \geq h \,. \qquad (1)$$

In this work, the uncertainty relation (1) is verified experimentally for photons. In optics the principle of uncertainties manifests itself for instance in the phenomenon of diffraction. Indeed, when the transverse dimensions of the light beam are limited by a slit of width Δx, the uncertainty of the coordinates of the photons which form the beam is equal to the width of this slit. Then the uncertainty of the photon momentum projection perpendicular to the slit isn't equal zero. Thus, there are photons in the diffracted radiation, which move not only in the initial direction, but also at a certain angle α to it. The projection of the momentum of such photons in the direction perpendicular to the direction of the original beam will be equal $p_x = p \cdot \sin\alpha$. If we consider that due to the diffraction the main fraction of the radiation is concentrated in the range of angles from $-\alpha$ to α, then the uncertainty of the momentum will be $\Delta p_x = p \cdot \sin\alpha = \frac{h}{\lambda}\sin\alpha$, where λ is the radiation wavelength.

Consequently, the product of the uncertainties of the coordinate and momentum will be equal to

$$x \cdot \Delta p_x = \frac{h}{\lambda}\sin\alpha \cdot \Delta x \geq h \,. \qquad (2)$$

From this inequality it follows that the angular divergence of the light beam after diffraction by a slit of width Δx is determined by the inequality $\sin\alpha \geq \frac{\lambda}{\Delta x}$. This corresponds to the classical theory of the diffraction, in which the angle of diffraction divergence of a beam is $\sin\alpha = \frac{\lambda}{\Delta x}$. Here, the angle α determines the directions to the first minima of the diffraction pattern.

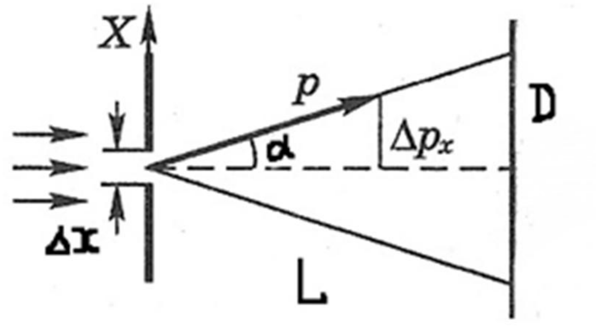

Fig. 9.1

As can be seen from Figure 9.1, for small angles $sin\,\alpha \approx tg\,\alpha = D/L$, where D is the half width of the main maximum of the diffraction pattern on the screen, which is located at a distance *L* from the slit. Therefore, relation (2) takes the form $x \cdot \Delta p_x = \frac{h}{\lambda} sin\,\alpha \cdot \Delta x = \frac{hD}{\lambda L} \cdot \Delta x \geq h$, whence it follows that

$$\frac{D \cdot \Delta x}{\lambda L} \geq 1 . \qquad (3)$$

Inequality (3) is easy to check by an experiment.

In this work it's proposed to measure the slit width, which characterizes the uncertainty of the photon coordinate Δ*x*, and the half-width of the diffraction pattern D, which characterizes the uncertainty of the photon transverse momentum Δp_x, and then verify the inequality (3) at different values of the slit width.

DESCRIPTION OF THE EXPERIMENTAL SETUP

The experimental setup for verification of the uncertainty principle (Fig. 9.2) consists of a source of monochromatic radiation (helium-neon laser) *1* and an optical bench *2* on which a variable-width slit *3*, a screen with a scale *4*, and a calibration microscope *5* are located.

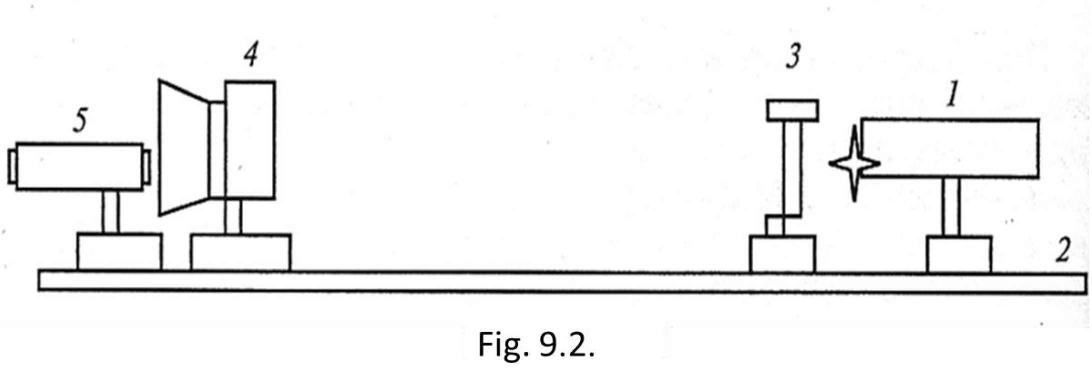

Fig. 9.2.

MEASUREMENTS AND DATA PROCESSING

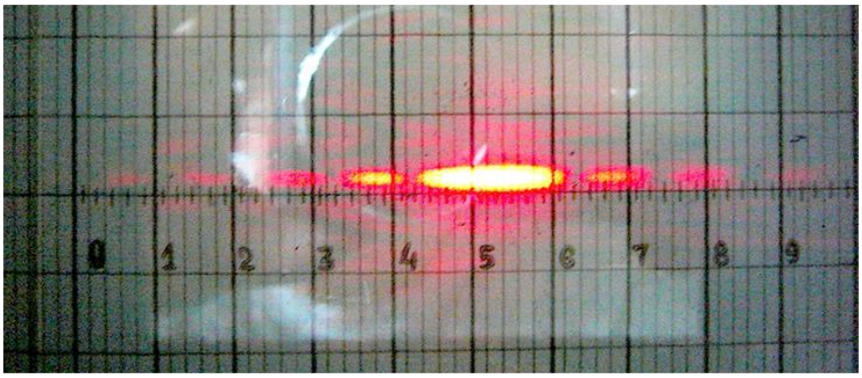

Fig. 9.3.

A beam of light emitted by a helium-neon laser passes through a slit and hits the screen, where the diffraction pattern is observed (Fig. 9.3). Changing the slit width with the help of a reel, one can observe a change in the diffraction pattern.

Task 1. **Measurement of the width dependence of the main diffraction maximum on the width of the slit**

1. Remove the calibration microscope from the optical bench, install the slit and screen according to Fig.9.2 at a distance exceeding 1 m.
2. Turn on the laser.
3. Get a diffraction pattern on the screen. To do this, use the slider on which a calibrated slit is installed and achieve that the laser beam passes through its hole and hits the screen.
4. Changing the slit width Δx with a step of 2 divisions on the reel scale, make 6-8 measurements of the $2 \cdot D$ width of the main maximum of the diffraction pattern from the value, when the diffraction pattern is already clearly visible on the screen, to a value at which one can still observe a change in the width of the main maximum.
5. Measure the width of the main maximum using the screen scale. To increase measurement accuracy, place the screen at a distance of $L \approx 1.0$ m from the calibrated slit. The width of the maximum is determined by the position of the dark bands bordering a maximum.
6. The measurement results of Δx (in divisions of the reel scale), $2 \cdot D$ and D (in mm) (half the width of the main maximum), as well as the distance L between the screen and the slit are in Table 1.

Table 1

№ experiment	**Δx**, divisions	**Δx,** mm	2**D,** mm	**D,** mm	**L,** cm	$F = \dfrac{D \cdot \Delta x}{\lambda L}$
1						
2						
…….						

T a s k 2 . **Calibration of the slit width**

To associate the number of divisions counted on its reel with the width of the corresponding slit, it is necessary to calibrate the slit using a microscope (type MPB-2).

1. Turn off the laser.
2. Place a slit and a microscope on the optical bench according to Fig. 9.4.
3. Close the slit. Open it to the width corresponding to a single division of the ocular scale (which is 0.05 mm).
4. Note the reel number.
5. Increase the slit width by 1 division of the ocular scale noting the reel readings. Proceed up to the width of about 0.4 mm. Make a calibration plot.
6. Using the plot, fill in the corresponding column in the Table 1.

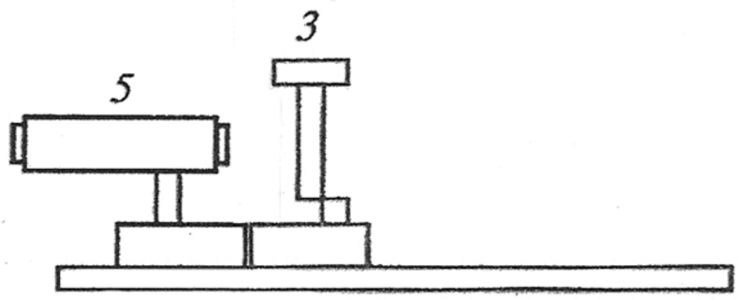

Fig. 9.4.

T a s k 3 . **Verification the uncertainty relation**

1. Plot the dependence of the half-width of the main maximum D on the size of the gap Δx.
2. Calculate the dimensionless quantity $F = \dfrac{D \cdot \Delta x}{\lambda \cdot L}$, where $\lambda = 6.33 \cdot 10^{-7}$ m is the wavelength of the emitted light. Make sure that this value practically does not change with Δx.
3. Verify the inequality (3).

QUESTIONS AND EXERCISES

1. What was the de Broglie hypothesis? What experiments have confirmed this hypothesis? What are the phase and group velocities of de Broglie waves? What is the de Broglie wavelength?
2. What is the physical meaning of the uncertainty relation?
3. Consider an imaginary experiment on the diffraction of electrons on two slits. What would be observed on the screen in the case of classical particles? Classic waves? What is actually observed? Is it possible to find out in which gap an electron has passed, while maintaining the diffraction pattern?
4. What advantages does the analysis of the value $F=\Delta x D/\lambda L$ provide in comparison with the analysis of the dependence $D=f(\Delta x)$?
5. Why is the verification of the uncertainty relation with the help of laser radiation more reliable than when working with other light sources (for example, a discharge lamp)?
6. What is zero oscillation? How to explain the presence of zero oscillations using the uncertainty principle?
7. Using the uncertainty relation, estimate: a) the energy of the ground state of a particle “locked” in a one-dimensional potential box of length l; b) the ground state energy of a one-dimensional oscillator; c) the size of the hydrogen atom and its energy in the ground state.
8. What is the natural width of the spectral line? What explains the additional broadening of lines? Using the uncertainty relation, estimate the blurring of the energy level in a hydrogen atom: 1) for the ground state 2) for the excited state.
9. Why can an electron microscope provide a higher resolution than an optical microscope?