ATOMIC PHYSICS
BLACK-BODY RADIATION

Practical 4
STUDY OF THERMAL RADIATION LAWS

1 Introduction

Electromagnetic radiation of heated bodies is called thermal radiation. The spectral characteristic of thermal radiation of a body surface with a temperature *T* is the *emissivity* of the body:

$$\varepsilon(\lambda, T) = \frac{dW_{rad}}{d\lambda}, \qquad (1)$$

where dW_{rad} - is the energy of electromagnetic radiation emitted from a unit of the body surface in the wavelength interval from λ to $\lambda + \Delta\lambda$, per unit time.

The spectral characteristic of absorption is the *absorptive capacity* of the body

$$\alpha(\lambda, T) = \frac{dW_{abs}}{dW_{inc}}, \qquad (2)$$

which shows the amount of the energy *dW* of electromagnetic waves (in the wavelength range from λ to $\lambda + \Delta\lambda$) falling per unit surface area of the body per unit time is absorbed by the body. Often, instead of $\varepsilon(\lambda,T)$ and $\alpha(\lambda,T)$, we introduce $\varepsilon(\nu,T)$ and $\alpha(\nu,T)$.

Experiment shows that the emissivity and absorptive capacity of a body depends on the wavelength of the irradiated and absorbed waves, on the body temperature, on its' chemical composition and on the state of the surface of the body. A body is called a perfect black body if it at any temperature completely absorbs all the energy of the electromagnetic waves incident on it, regardless of the frequency. The absorptive capacity of a perfect black body is always equal to unity, while the emissivity $\varepsilon_0(\lambda,T)$ depends only on the wavelength λ, and the absolute temperature *T* of the body.

According to Kirchhoff's law, the ratio of the emissivity of any body in thermal equilibrium with radiation to its absorptive capacity in a narrow wavelength interval (λ, $\lambda + \Delta\lambda$) does not depend on the material of the body and is equal to the emissivity of a perfect black body $\varepsilon_0(\lambda,T)$:

$$\frac{\varepsilon(\lambda, T)}{\alpha(\lambda, T)} = \varepsilon_0(\lambda, T). \qquad (3)$$

According to the Stefan-Boltzmann law, the integral emissivity of a black body (in other words, the energy emitted by a unit of the body surface per unit time in the entire frequency range) is proportional to the fourth power of the body temperature:

$$\varepsilon_0(T) = \int_0^\infty \varepsilon_0(\lambda, T)\, d\lambda = \sigma T^4, \qquad (4)$$

where $\sigma = 5.67 \times 10^{-8}$ W/(m^{2}K^4) - is the Stefan-Boltzmann constant.

For real bodies the ratio of the integral emissivity to the integral emissivity of a black body at the same temperature is less than unity. Fig. 1 shows how this ratio varies depending on the temperature of tungsten - the metal from which the filament of the incandescent lamp is made. Under real conditions, the power $I \times V$ going to heat the filament, whose surface area is equal to *S*, is almost completely transferred to the surrounding space in the form of thermal radiation. Then

$$I \times V = \alpha(T)\sigma T^4 S, \qquad (5)$$

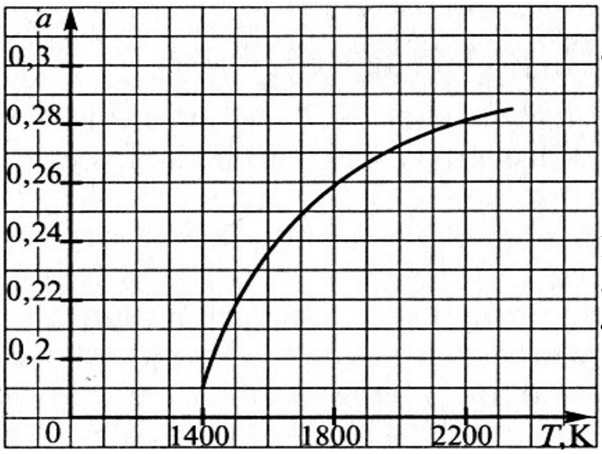

Fig. 1. The ratio of the integral emissivity to the integral emissivity of a black body at the same temperature for tungsten.

This equation enables the experimental determination of the Stefan-Boltzmann constant:

$$\sigma = \frac{I \times V}{\alpha(T) T^4 S}. \qquad (6)$$

The search for the explicit form of the function $\varepsilon_0(\lambda, T)$ led to the determination of the quantum nature of radiation and energy absorption by atoms and molecules. The function $\varepsilon_0(\lambda, T)$ obtained by Planck has the form:

$$\varepsilon_0(\lambda, T) = \frac{2\pi h c^2}{\lambda^5}(e^{\frac{hc}{\lambda kT}} - 1)^{-1}, \qquad (6)$$

where h = 6.625 × 10^{-34} J·s – is the Planck constant, c = 299792458 m/s – is the speed of light in vacuum; k = 1.38 × 10^{-23} J/K - is Boltzmann's constant.

The spectral emissivity of a real metal can be obtained by multiplying $\varepsilon_0(\lambda,T)$ by the absorptive capacity of the metal $\alpha(\lambda,T)$: $\varepsilon(\lambda,T) = \alpha(\lambda,T) \times \varepsilon_0(\lambda,T)$.

The ratio of the spectral emissivity of a metal for different wavelengths λ_1 and λ_2 at the same temperature is:

$$\varepsilon_{12} = (\frac{\varepsilon_1}{\varepsilon_2}) = (\frac{\lambda_2}{\lambda_1})^5 \frac{\alpha_{1T} e^{\frac{hc}{kT\lambda_2}} - 1}{\alpha_{2T} e^{\frac{hc}{kT\lambda_1}} - 1}. \qquad (7)$$

Taking into account that 1) wavelengths lying in the visible part of the spectrum and 2) temperatures exceeding room temperature, one can easily find that ratio $e^{\frac{hc}{kT\lambda}} - 1$ considerably exceeds unity, It is easy to obtain from equation (6) the ratio of the spectral spectral emissivity of the metal at different temperatures T_1 and T_2:

$$R = \frac{(\varepsilon_{12})T_1}{(\varepsilon_{12})T_2} = \frac{(\alpha_1/\alpha_2)T_1}{(\alpha_1/\alpha_2)T_2} \exp((\frac{hc}{k})(\frac{1}{\lambda_2} - \frac{1}{\lambda_1})(\frac{1}{T_1} - \frac{1}{T_2})). \qquad (8)$$

Graphs of the dependence of $\alpha(\lambda,T)$ on the temperature T (in the range 1700-2500 K) for tungsten for two different wavelengths are shown in Fig. 2.

It can be seen from Fig. 2 that the ratio $\frac{(\alpha_1/\alpha_2)T_1}{(\alpha_1/\alpha_2)T_2}$ is close to unity. Therefore, if the ratio of the measured emissivity on the left-hand side of expression (6) is equal to unity, then from equation (8) it is possible to calculate Planck's constant:

$$h = \frac{\gamma \ln(R)}{1/T_1 - 1/T_2}, \qquad (9)$$

where

$$\gamma = \frac{k}{c(1/\lambda_2 - 1/\lambda_1)}. \qquad (10)$$

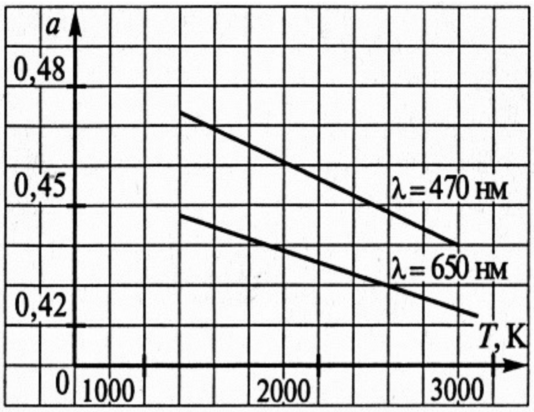

Fig. 2. α(λ,T) dependence on the temperature T (1700-2500 K) for tungsten.

2 Experimental setup

The setup is assembled according to the schematic shown in Fig. 3b. The general view of the setup is shown in Fig.3a.

In this work the light-measuring lamp СИ-6-100 with a tungsten filament of the area *S* is used to study thermal radiation. The lamp is fixed in a holder and surrounded by a metal screen with a window.

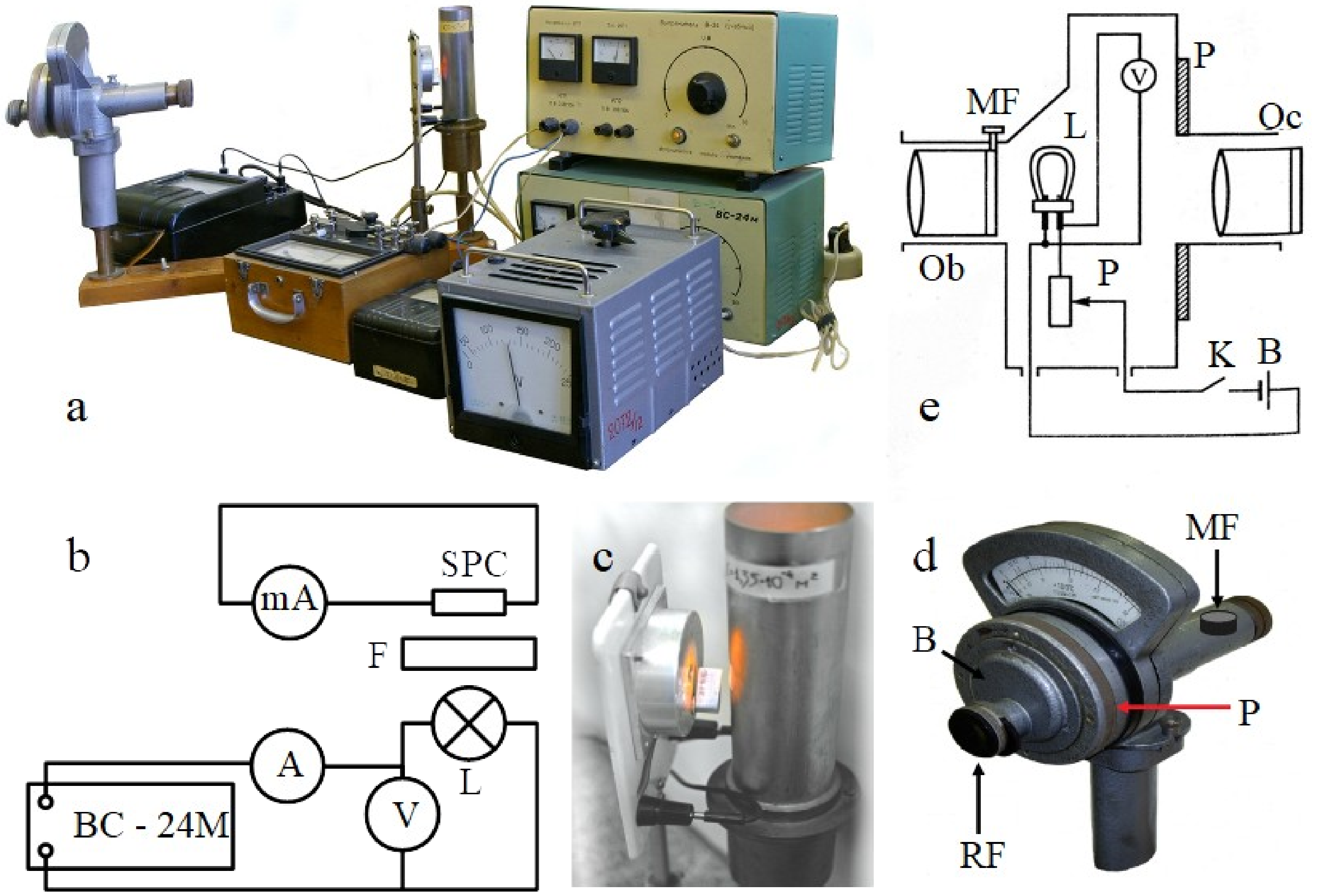

Fig . 3.a) A general view of the setup; b) A schematic of the setup; c) The light filter control system; d) A general view of the optical pyrometer; e) A schematic of the optical pyrometer.

The voltage to the lamp is supplied from the semiconductor rectifier BC-24M, which is connected to the mains via the transformer «ЛАТР». During the measurements, the potentiometer of the BC-24M rectifier must remain in the fixed position (marked with the pointer). The voltage of the lamp is changed by turning the voltage regulation knob on the top panel of the laboratory transformer. The power required to heat the tungsten filament is determined by a voltmeter and an ammeter.

The interchangeable blue and red filters with average wavelengths of λ_{blue} = 460 nm, λ_{red} = 610 nm are used in this practical, to derive the thermal radiation corresponding to a certain wavelength interval.

The light filters are located in the frame, which mounts in front of the selenium photocell - *SPC*, which is fixed in the holder (Fig. 3c). These light filters are placed in the path of the light flux during the measurement.

The sensitivity of the photocell to blue and red light is almost identical. Therefore, the ratio of photocurrents in the photocell chain with red (I_{red}) and blue (I_{blue}) light filters will depend on the temperature in the same way as the ratio of the spectral emissivity in these wavelength intervals. Thus, the ratio of photocurrents can be used to estimate the ratio of the spectral emissivities of tungsten in the selected wavelength intervals. It is recommended to use multi-limit ammeters, to measure the photocurrents.

A photometric comparison of the brightness of the studied body (the filament of the lamp being investigated) and the reference lamp should be used in this practical to determine the temperature of the studied body. Such approach involves the usage of an optical pyrometer.

An optical pyrometer with a fading filament consists of a body - *B*, in which a standard incandescent lamp - *L* with a bent filament is placed (Fig. 3d and 3e).

By moving the pyrometer objective - *Ob*, one can obtain an image of the tungsten filament in the plane of the reference lamp filament. To obtain a sharp image of the filament of the reference lamp and the image of the studied filament in the same plane, one needs to move the eyepiece - *Oc*. The reference lamp through the key *K* is powered by a current from a battery *B* or from a stabilized source (V = 3 V). The temperature of the reference filament is regulated by a potentiometer *P* by means of a ring *P* located in the front part of the pyrometer.

When measuring the temperature of the heated body, one needs to adjust the current of the pyrometer reference lamp (potentiometer ring *P*) until the brightness of the filament of the reference lamp coincides with the brightness of the studied body (the upper part of the arc of the reference lamp filament disappears against the background image of the tungsten filament of the lamp - see Fig. 4.

The temperature of the filament corresponding to the temperature of a perfect black body in monochromatic light with a wavelength of λ = 650 nm (the red filter *RF* is placed in front of the eyepiece of the pyrometer, which can be removed if necessary by rotating the ring on the eyepiece), is counted with a voltmeter with a scale graduated in degrees Celsius.

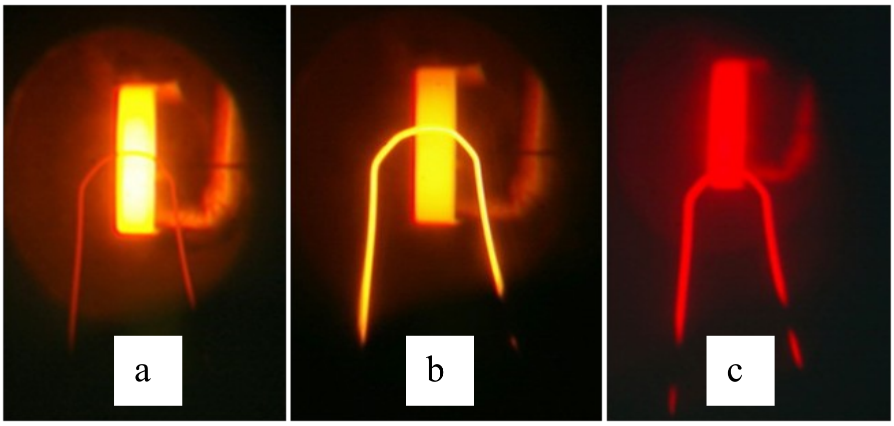

Fig 4. An example of the image of the reference filament against the background image of the tungsten filament: a – underheated reference filament; b – overheated reference filament; c - The temperature of the filament corresponding to the temperature of a perfect black body in monochromatic light (λ = 650 nm).

In the pyrometer there are two graduated scales: from 0 to 1400 ° C (lower scale) marked with a blue dot; from 1400° to 2000 °C (upper scale) marked with a red dot. When measuring temperatures above 1400 ° C, a matte filter - *MF* must be used (Fig. 3d). To place a matte filter, one needs to align the point indicator on the *MF* head with the red dot on the pyrometer case, then use the objective lens to adjust the focal distance and again use the upper scale. While measuring the temperature from 0 to 1400 ° the matte filter is not used, therefore, the reference point on the *MF* head must be aligned with the blue dot on the pyrometer case.

On the pyrometer scale, the so-called brightness temperature T_B of the body is measured. Here, the brightness temperature is understood as the temperature of a perfect black body, at which the emissivity of a perfect black body is equal to the emissivity of the body under study at its physical temperature in a single wavelength interval.

There is a relationship between the brightness - T_B and the physical - T temperature of the body:

$$\frac{1}{T} = \frac{1}{T_B} + \frac{\lambda k}{hc} \ln \alpha_{\lambda T}, \qquad (11)$$

where λ = 650nm. This dependence is shown in the graph attached to the setup and is used to determine the physical temperature T of tungsten from the measured values of the T_B.

3 Measurement and data processing

Recommendations for the experiment:

1. The lamp voltage is supplied as follows:

• Plug in the laboratory transformer «ЛАТР» in the power supply network;

• Turn on the rectifier ВС-24М with the switch «On/Off» (**do not turn the voltage adjustment knob !!!**);

• Set the voltage output knob to the far left position, then switch the switch «On/Off» on the laboratory rectifier «ЛАТР» to the ON position ;

• by slowly rotating the voltage control knob, set the current value on the lamp equal to 7.0 A.

2. Measurement of the brightness temperature of the filament of the lamp:

• The brightness temperature of the filament is measured within the range of currents from 6.5 up to 9.0 A, s written in the Table 1.

• The measurement of the filament temperature is performed with the lower scale of the pyrometer without the matte filter for the current range from 6.5 up to 8.5 A;

• The measurement the filament temperature is performed with the upper scale of the pyrometer with the matte filter for the current range from 8.5 up to 9.0 A (align the point indicator on the *MF* head with the red dot on the pyrometer case to place the matte filter on the objective).

Task 1. Determination of the Stefan-Boltzmann constant.

1. Before starting the measurements, make sure that when placing in sliders: the filter window in front of the selenium photocell *SPC*, the filament of the studied lamp, the input window and the filament of the pyrometer are approximately on the same line.

2. Remove the photocell *SPC* from the optical bench.

3. Suppling a small voltage (corresponding to the current of 6,7 A) to the СИ-6-100 lamp, get a sharp image of the filament of the lamp in the plane of the pyrometer eyepiece.

4. Rotating the potentiometer knob of the laboratory transformer «ЛАТР» slowly increase the voltage on the lamp.

Measure at 8 different voltages at which the current in the lamp varies from 6.7 up to 8.8 A (see Table 1). For each measurement, proceed as follows: set a certain current value, wait for 3 minutes, then measure the brightness temperature of the tungsten filament.

5. Find the physical temperature of the filament using the graph given in Fig. 1.

6. For each value of the temperature of the studied filament find the value *α(T)* using the graph given on Fig. 2.

7. Set the photocell perpendicular to the light coming from the lamp, and for each current value on the lamp (see table 1), measure the current in the photocell circuit with the red (λ_{red} = 610 nm) and blue (λ_{blue} = 460 nm) light filters.

Switch the filters by turning the handle on the photocell cap. When measuring currents, use different range limits of the ammeter.

8. For each power absorbed by the lamp, calculate the Stefan-Boltzmann constant (the area of the emitting surface of the tungsten ribbon $S = 1.35 \times 10^{-4}$ m^2) using the formula (6).

9. Put down the results of the measurement and calculation of I, U, I_{red}, I_{blue}, T_B(K), T_B(°C), T(K), $\alpha(T)$ into table 1.

10. Calculate the mean value of the Stefan-Boltzmann constant σ.

11. Using the tabulated value of the Stefan-Boltzmann constant, estimate from a handbook the accuracy of your measurements.

Table 1.

#	1	2	3	4	5	6	7	8
I, A	6.7	7.0	7.3	7.6	7.9	8.2	8.5	8.8
V, V								
T_B, °C								
T_B, K								
T, K								
$\alpha(T)$								
σ_{exp}								
I_{red}, μA								
I_{blue}, μA								
$(\varepsilon_{12})_T = I_{red} / I_{blue}$								

Task 2. An estimate of Planck's constant.

1. Calculate the quantity γ that does not depend on temperature using formula (10), while assuming $\lambda_1 = \lambda_{red}$ and $\lambda_2 = \lambda_{blue}$. The values of the remaining constants should be taken from the physical handbooks.

2. Using the measurement results obtained in the task 1 and formula (9), calculate Planck's constant for five temperature combinations at which measurements were taken (for example, temperature combinations for the following pairs of measurements: *i-k*: 3-6, 3-7, 4-8; 5-7; 5-8). Put down the calculation data into table 2.

Note: with increasing temperature, the ratio $(\varepsilon_{12})_T = I_{red} / I_{blue}$ should decrease; those columns of the Table 1 for which this is not satisfied can not be used to calculate Planck's constant !!!

3. Using the value of Plank's constant from a handbook, estimate the accuracy of the obtained results.

Table 2.

i-k					
$1/T_i$, K^{-1}					
$1/T_k$, K^{-1}					
$\frac{1}{T_i} - \frac{1}{T_k}$					
$R = \frac{(\varepsilon_{12})_{T_i}}{(\varepsilon_{12})_{T_k}}$					
ln R					
h, (J·s)					

4 Questions

1. What is thermal radiation? What is the difference between thermal radiation and other types of radiation?

2. Give definitions of the following: energy luminosity (or integral emissivity), spectral emissivity (or spectral density of radiation), absorbion capacity.

3. What is the idea of introducing the concept of "a perfect black body"? Describe the experimental model of such a body, proposed by Kirchhoff. Are there such bodies in nature? What are the "grey" bodies?

4. Formulate the basic laws of thermal radiation.

5. What temperature should a perfect black body have, so that the maximum emissivity $\varepsilon_0(\lambda,T)$ corresponds to the "red" region of the spectrum of visible radiation?

6. Does the Wien displacement law applicable to all radiating bodies?

7. Draw a graph showing the energy distribution in the emission spectrum of a perfect black body. How will this graph change with increasing temperature?

8. What are contradictions between the experimental data and the classical theory of a black body radiation? What is the "ultraviolet catastrophe"?

9. Write down Planck's formula for the spectral emissivity of a perfect black body as a function of frequency and as a function of wavelength. Plot a family of curves for the spectral emissivity of a perfect black body as a function of frequency and as a function of wavelength for different temperatures. What is the physical meaning of the area under the curve? How does the maximum of these functions change with T?

10. Using Planck's formula, prove that in the low-frequency region ($h\nu << kT$) it coincides with the Rayleigh-Jeans formula, while in the high-frequency region it follows ($h\nu >> kT$)) the Wien formula.

11. Explain the origin of radiation, color and brightness temperatures. Which of them is measured by a pyrometer with a vanishing filament? Describe the operation principle of such an optical pyrometer and the methodology of its use in the practial.

12. Derive formula (11), which connects the brightness temperature and physical temperature of the body.