Objective: experimental verification of the applicability of the Bernoulli Equation for the case of the water flow within a tube with variable cross section.

Equipment: Bernoulli tube, stopwatch, beaker.

INTRODUCTION

Steady state flow of an ideal fluid through a sufficiently thin horizontal tube (the fluid velocity at any point of the tube cross section is the same) can be described by Bernoulli’s Equation:

\[ p_1 + \frac{\rho v_1^2}{2} = p_2 + \frac{\rho v_2^2}{2}, \]  

(1)

where \( \rho \) is the fluid density, \( p_1, v_1, p_2, v_2 \) are the values of pressure and velocity for two arbitrary cross section of the tube (\( S_1 \) and \( S_2 \)). The flow rate (namely, the amount of fluid that flows through a arbitrary cross section of the tube per unit time) can be determined as:

\[ Q = \frac{2(p_2 - p_1)}{(\frac{1}{S_1} - \frac{1}{S_2}) \rho}. \]  

(2)

In this work we study the flow of water within a horizontal tube with variable cross section. Water is not an ideal fluid, and thus the experimentally determined dependence of the flow rate from the pressure will differ from the theoretical dependence predicted by Eq. (2).

EXPERIMENTAL SETUP

The tube with variable cross-section is shown in Figure 20.1. The tube is connected with the tap at one end. Water flows through the tube from the tap to a sink or a measuring beaker. Two vertical pipes are soldered in the narrow \( S_1 \) and wide \( S_2 \) sections of the
horizontal tube. The difference in water levels in these pipes shows a difference in water pressure in the horizontal cross sections of tube. The upper ends of pipes are connected together with a rubber hose, which falls into the sink. This prevents the uncontrolled release of water through the tube at high level of pressure difference $\Delta p$. The flow rate of water is regulated by the tap and can be determined by using the beaker.

**MEASUREMENT AND DATA PROCESSING**

The experimental setup allows to measure the flow rate for a given pressure difference $\Delta p = p_2 - p_1$ between two fixed pipe sections. The flow rate $Q$ is determined by measuring the volume of water $V$, filling the measuring beaker during the time $\Delta t$.

*Experimental task.* Determine $Q$ for five different values of $\Delta p$. You should start from the value of $\Delta p = 20$ mm and continue with the step of 20 mm on the water column. It should make at least three series of such measurements. Plot the graph for the given data $Q = f(\sqrt{\Delta p})$. Plot at the same graph the theoretical dependence expressed by the Eq. (2). Evaluate the errors of the method. Put the all measured ($V$, $\Delta t$, $Q_{\text{meas}}$, $\Delta p$) and calculated data ($Q_{\text{theor}}$) in the Table.

**QUESTIONS AND EXERCISES**

1. Explain difference between theoretical and experimental values of $Q$.
2. Will the experimental results change if you change the direction of the water flow?
3. How can you measure static and dynamic pressure in the tube cross section?
4. What factors determine the random errors in the experiment? How can you improve the accuracy of the measurement?
5. Prove the formula (1) and (2).
PRACTICAL 1.22

DETERMINATION OF VISCOSITY COEFFICIENT OF LIQUID BY THE METHOD OF STOKES

Objective: find coefficient of viscosity of glycerol.

Equipment: cylindrical vessel with glycerol, lead pellets, ruler, micrometer, stopwatch.

INTRODUCTION

The time dependence of a ball-shaped body velocity in a viscous liquid is described by the expression

$$v = \frac{2}{9} \frac{g(\rho - \rho_i)r^2}{\eta} \left(\frac{t}{1 - e^{-\frac{t}{\tau}}}\right)$$

(1)

where $\rho, \rho_i$ — density of the ball and liquid, $r$ — radius of the ball, $g$ — acceleration of gravity, $\eta$ — the coefficient of viscosity of the liquid, $\tau = \frac{2}{9} \frac{\rho r^2}{\eta}$ — the setting time of the movement.

At $t \to \infty$ (when $t >> \tau$) the motion becomes uniform. This formula is valid for laminar movement of the liquid relative to the ball. In this case the friction force $F$ is described by the Stokes formula $\bar{F} = -6\pi r \eta \bar{v}$. The motion can be considered laminar if the Reynolds number $Re = \rho_i v r / \eta < 2300$.

In the method of Stokes, the viscosity of the liquid is determined from the measurement of the ball size and steady-state velocity $v_{st}$.

$$\eta = \frac{2}{9} \frac{g(\rho - \rho_i)r^2}{v_{st}}$$

(2)

EXPERIMENTAL SETUP

For the experiment, a cylindrical glass vessel filled with glycerol is used. Lead pellet is used as the ball-shaped body. The distance between two ring-shaped marks can be measured with a ruler. First mark should be chosen such that the pellet motion has already become uniform, and the diving speed does not change. The time of the pellet motion between marks is measured with a stopwatch. The radius of the pellet is measured with a micrometer. To prevent air bubbles sticking to the lead pellet it should be moisten with the glycerol first.
**MEASUREMENTS AND DATA PROCESSING**

**Task/Assignment. Determination of the coefficient of viscosity of the glycerol**

Measure the speed of the pellet. Execute the experiment five times, tabulating all the variable parameters, \( t, l, v_0, r \). Calculate the value of \( \eta \) in each experiment. Define the average value of \( \eta \) and the maximum value of \( \Delta \eta \). The values of \( \rho \) and \( \rho_i \) find out in the reference tables. Calculate the Reynolds number for the experiments. Calculate the setting time of the uniform motion \( \tau \) for the experiments. Form a table with the measured and calculated data.

**QUESTIONS AND EXERCISES**

1. Write down the equation of motion of a pellet in a viscous liquid. Find its solution and derive expression (1) and \( \tau \).

2. The pellet falls on the surface from the height \( h \). Find pellet's speed at the liquid surface and derive corrected expression (1).

3. Explain the physical meaning of the Reynolds number.

4. Write down the equation of motion of a pellet thrown upward with an initial velocity \( v_0 \), if the motion is in a viscous liquid. Find its solution.

5. Why the time countdown of motion of the pellet is not taken from the surface of the liquid, but from a certain depth (indicated by the first mark)? How to assess this depth?

6. Using the dependence of viscosity on concentration of water in glycerol \( \eta(k) \) find \( k \) for the glycerol in your experiment. Find and use \( \eta(k) \) from the reference tables. Try linear and spline interpolation.